Holographic directivity measurement of line sources and sound panels

Christian Bellmann, ICSA 2019 Ilmenau
Abstract

Compared to a standard Hifi stereo system, new 3D listening experiences have increased the complexity of sound systems a lot. To realize immersive audio reproduction, many loudspeakers distributed in the listening room may be placed. Other methods use more complex sound sources like line arrays and sound panels to shape distinct beams, including controlled reflections from the room boundaries. The complex control algorithms necessitate directivity data of each individual transducer with highly accurate phase information.

The workshop will discuss the field of directivity measurement for multi-transducer sound systems and will compare the benefits and limits of a traditional far field measurement vs. new holographic measurement techniques.
Reproducing 3D sound

Distributed sound sources

- Sound sources are distribute in the listening room
- Fixed installation
- Sound sources are relatively simple

Centralized audio system

- Sound system is at one spot in the listening room
- (e.g. TV, stage, etc.)
- Spatial perception generated by controlled reflections
- portable
- high complexity of the sound source
- many transducers, large dimension
Controlled directivity

DSP / Smart Amplifier

Delay

Filter

right

left

center

Holographic directivity measurement
Example: Sound Bar

Home Cinema application
- Distinct beams for front channels
- Rear channel by controlled reflections

Holographic directivity measurement

©sennheiser.com
Measurement Requirements

Targets:
• Directional characteristics of each transducer
• Including boundary effects from the cabinet
• Far field (Pro Audio Line Arrays)
• Near Field (e.g. sound bars, studio monitor)
• Accurate Phase information

Measurement Particularities
• Free field data
• Separate measurement of transducers
• Positioning of sources is critical
• Measurement distance? Near Field or Far Field?
• Sound radiation problems (e.g. humidity, temperature)
• Reasonable Time, minimum number of points
Measurement of Far Field Response

Voltage Spectrum at Terminals

Sound Pressure spectrum

Magnitude of transfer function $H(f)$

Phase of transfer function $H(f)$

Impulse response $h(t)$

Distance $> 1$ m

Shaped Stimulus

Stimulus ($t$) vs time

Impulse response windowing

Fourier Transform

$H(j\omega) = \frac{P(j\omega)}{U(j\omega)}$
Extrapolation of Far Field Data

Extrapolation Not applicable

\[ H(f, r_2, \theta, \phi) = H(f, r_1, \theta, \phi) \frac{r_1}{r_2} e^{-jk(r_2-r_1)} \]

Requirements:
- free field condition (direct sound)
- far field condition
- same direction \((\phi_2 = \phi_1, \theta_2 = \theta_1)\)
How to Ensure Far-Field Conditions?

Requirements:

- **Distance** $r_{\text{far}} >> d$
  (critical for large geometrical dimension $d$)

- **Distance** $r_{\text{far}} >> \lambda$
  (critical at long wavelength $\lambda$)

- **ratio** $r_{\text{far}}/d >> d/\lambda$
  (critical at short wavelength $\lambda$)

→ Large loudspeaker systems require large anechoic rooms! (e.g. line arrays)
Conventional Far-Field Measurements

(a time line)

• Far-Field Measurements in Anechoic Chambers
  (1930’s, Beranek and Sleeper 1946)
  - Realized as a half and full space
  - Good absorption of room reflections (> 100 Hz)
  - High ambient noise isolation
  - Controlled climate conditions and avoids wind effects

• Far-Field Measurement under simulated free-field conditions by gating or windowing the impulse response (Heyser 1967-69, Berman and Fincham 1973)
  - Good suppression of room reflections at higher frequencies
  - Higher SNR due to ambient noise separation
  - Limited low frequency resolution (depends on time difference between direct sound and first reflection)
Time Windowing

Holographic directivity measurement
**Time Windowing**

\[ d_r = 2 \sqrt{\left(\frac{1}{2} d_{ms}\right)^2 + d_{sr}^2} \]

\[ T_w = \frac{d_r - d_{ms}}{c} \]

\[ \Delta f = \frac{1}{T_w} \]

**Example:** \( d_{ms} = 4 \text{m} \)

<table>
<thead>
<tr>
<th>( d_{sr} )</th>
<th>( T_w )</th>
<th>( \Delta f )</th>
<th>1/12 octave</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 m</td>
<td>4.8 ms</td>
<td>200 Hz</td>
<td>&gt;1 kHz</td>
</tr>
<tr>
<td>5 m</td>
<td>19.7 ms</td>
<td>50 Hz</td>
<td>&gt;500 Hz</td>
</tr>
</tbody>
</table>

- Room size is limiting the Frequency Resolution
- Not applicable for low frequencies

\( d_{ms} \) - Distance microphone speaker
\( d_{sr} \) - Distance speaker room boundary
Angular Resolution limited by Sampling

Problem of the Far Field Measurement

The sound pressure is measured at multiple measurement points located on a sphere with radius r. The # of pts. depends on desired resolution:

- 5 degree $\rightarrow$ 2592 points
- 2 degree $\rightarrow$ 16200 points
- 1 degree $\rightarrow$ 64800 points

Not practical

Accuracy of measurement depends on:
- tolerance of microphone placement (both $\theta$ and r)
- DUT positioning while maintaining the acoustic center
- Sound reflections from turntable
- Room absorption irregularities
Directional Far Field Characteristics

Far-Field Sound Pressure
\[ p(r, \theta, \phi) \]

Sound Pressure On-Axis
\[ p_{ax}(r) = p(r, \theta = 0, \phi = 0) \]

SPL On-Axis
\[ SPL_{ax}(r) = 20 \log \left( \frac{p(r,0,0)}{p_0} \right) dB \]
With \( p_0 = 20 \mu Pa \)

Directional Gain
\[ D(\theta, \phi) = 20 \log \Gamma(\theta, \phi) dB \]

Directional Factor
\[ \Gamma(r, \theta, \phi) = \frac{p(r, \theta, \phi)}{p_{ax}(r)} \]

Directivity Factor
\[ Q = \frac{p_{ax}^2(r)}{p_0^2} = \frac{S}{\int \Gamma^2(\theta, \phi) dS} \]

Directivity Index
\[ DI = 10 \log_{10}(Q) dB \]

Sound Power
\[ \Pi = \frac{1}{\rho c} \int_{S} p(r, \theta, \phi)^2 dS \]
\[ = \frac{p_{ax}^2(r)}{\rho c} \int_{S} \Gamma(r, \theta, \phi)^2 dS \]
\[ = \frac{S}{\rho c} p_0^2(r) \]

Sound Power Level
\[ L_{11} = 10 \log_{10} \left( \frac{\Pi}{P_0} \right) dB \]
With \( P_0 = 10^{-12} W \)

Directional Gain 
\[ DI \approx SPL_{ax}(r = 0.4m) - L_{11} \]
Radiation into half space (using baffle)

Holographic directivity measurement
No anechoic room is perfect!
How to cope with limited absorption at low frequencies?

1. Select typical set of loudspeakers
2. Measure Loudspeaker in anechoic room and under free-field conditions
3. Calculate a room correction curve

Room correction curve depends on loudspeaker properties!!
Problems in the Far-Field

Phase response depends on air temperature

Speed of sound is dependent on air conditions (e.g. temperature)

- $\vartheta_1 = 20^\circ C \rightarrow c_1 = 343.4 \text{ m/s}$
- $\vartheta_2 = 22^\circ C \rightarrow c_2 = 344.6 \text{ m/s}$
- $\vartheta_3 = 24^\circ C \rightarrow c_3 = 345.8 \text{ m/s}$

A temperature difference of $\Delta \vartheta = 2^\circ C$ will change the speed of sound by $\Delta c \approx 1.2 \text{ m/s}$

Depending on the distance, the temperature difference will influence the sound wave propagation time:

Far Field Measurement

required measurement distance $> 5\text{ m}$

Deviation:

- $\Delta t = 0.05 \text{ ms}$
- $\Delta r = 17.2 \text{ mm}$

Phase error caused by temperature difference of $2^\circ C$ during

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Wave length</th>
<th>Phase Error in 5 m distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f=2\text{kHz}$</td>
<td>$\lambda=171.7 \text{ mm}$</td>
<td>$36^\circ (0.1 \lambda)$</td>
</tr>
<tr>
<td>$f=5\text{kHz}$</td>
<td>$\lambda=68.7 \text{ mm}$</td>
<td>$90^\circ (0.25 \lambda)$</td>
</tr>
<tr>
<td>$f=10\text{kHz}$</td>
<td>$\lambda=34.3 \text{ mm}$</td>
<td>$180^\circ (0.5 \lambda)$</td>
</tr>
</tbody>
</table>

Far field measurement are prone to phase errors!
Problems of conventional techniques

• Low frequency measurements (accuracy, resolution) limited by acoustical environment
• Large loudspeakers need large measurement rooms
• High frequency measurements require far-field conditions
• Accuracy of the phase response in the far-field depends on temperature deviations and air movement
• An anechoic chamber is an expensive and long-term investment which cannot be moved easily
• Far field is not relevant for near field applications
Sound Radiation
Far Field Characteristics

Frequency Response

Sound Power

Directivity Index

Contour Plot

Polar Plot

Directivity Balloon

2019 AES Conference on Headphone Technology - Workshop on 3D Directivity
Measurement Requirements

Targets:

- ✔ Directional characteristics
- ✔ Including boundary effects from the cabinet
- ✔ Far field (Pro Audio Line Arrays)
- ✗ Near Field (e.g. sound bars, studio monitor)
- ✗ Accurate Phase information,
- ✗ Reasonable Time
Measurements in the Near Field

Advantages:
- High SNR
- Amplitude of direct sound much greater than room reflections providing good conditions for simulated free field conditions
- Minimal influence from air properties (air convection, temperature deviations)

Disadvantages:
- Not a plane wave
- Velocity and sound pressure are out of phase
- $1/r$ law does not apply, therefore, no sound pressure extrapolation into the far-field (holographic processing required)

Solution ➔ Scanning + Holographic Postprocessing
Short History on Near-Field Measurements

Single-point measurement close to the source

Don Keele 1974

Multiple-point measurement on a defined axis

Ronald Aarts (2008)

Scanning the sound field on a surface around the source

Melon, Langrenne, Garcia (2009)
Bi (2012)

Robotics required

Postprocessing of the scanned data required

Holographic directivity measurement
Near Field Scanner

ARRANGEMENT AND METHOD FOR MEASURING THE DIRECT SOUND RADIATED BY ACOUSTICAL SOURCES

Klippel 2014
Measurement Setup

Near Field Scanner

Moving the microphone has the following advantages:

- Constant DUT interaction in the room during the scan (required in a non-anechoic environment)
- Accurate positioning of Mic
- Facilitate heavy loudspeakers (hanging on a crane)
- Minimum gear within the scanning surface (only a platform and a pole)
Sound Pressure Response

measured in a normal office

Front side (on axis)

Rear Side

Double layer scanning + holographic processing allows to separate the direct sound from room reflections

Holographic directivity measurement
Holographic Measurement Process

1st step: Near-field Scanning

2nd step: Holographic Data Processing

3rd step: Extrapolation

Holographic directivity measurement
2nd Step: Holographic Wave Expansion

General solutions of the wave equation are used as basic functions in the expansion:

Total number of coefficients = \((N+1)^2\)

3rd Step: Wave Extrapolation
How to Find the Maximum Order N?

The measurement system determines automatically:

- optimum order N of the wave expansion
- total number of the measurement points
- measurement time

Directivity at 2kHz:

- Target
- N=0
- N=1
- N=2
- N=5
- N=10

Holographic directivity measurement

Fitting error as a function of the maximum

bad SNR

Sufficient accuracy

Noise Floor

-20dB = 1%

Target

N=0

N=1

N=2

N=5

N=10
Wave Expansion of a Woofer

Directivity patterns at 200 Hz:

- Target
- N=0
- N=1
- N=2
- N=3
- N=10

**sound field is completely described by order N=3 (16 Coefficients)**

can be estimated by a few measurement Points (M > 16)

Holographic directivity measurement
Optimal Choice of the Expansion Point

Setting the expansion point to the center of the tweeter reduces the number of measurement points to 25%.
Expansion into Spherical Waves

The general solution of the wave equation in spherical coordinates is given by:

\[ p(r, \theta, \phi, \omega) = p_{\text{out}}(r, \theta, \phi, \omega) + p_{\text{in}}(r, \theta, \phi, \omega) \]

**Outgoing wave**

\[ p_{\text{out}}(r, \theta, \phi, \omega) = \sum_{n=0}^{N} \sum_{m=-n}^{n} c_{n,m}^{\text{out}}(\omega) h_n^{(2)}(kr) Y_n^m(\theta, \phi) e^{i\omega t} \]

**Incoming wave**

\[ p_{\text{in}}(r, \theta, \phi, \omega) = \sum_{n=0}^{N} \sum_{m=-n}^{n} c_{n,m}^{\text{in}}(\omega) h_n^{(1)}(kr) Y_n^m(\theta, \phi) e^{i\omega t} \]

- Coefficients \( c_{n,m}^{\text{out}}(\omega) \) and \( c_{n,m}^{\text{in}}(\omega) \) depending on frequency \( \omega \)
- Hankel functions \( h_n^{(1)}(kr) \) and \( h_n^{(2)}(kr) \) depending on distance \( r \)
- Spherical harmonics \( Y_n^m(\theta, \phi) \) depending on angular direction

**Region of Validity**

- External boundaries (walls)
- Surface
- Sound source
- External sound source (ambient noise)
Example: Studio Monitor

- Near-field scanning in an ordinary office room
- 500 points
- Order of expansion N=20

Holographic directivity measurement
Far Field – where does it start?

A useful characteristic for investigation the radial dependency of the sound pressure output is the apparent power

\[
\Pi_A(f, r) = \frac{1}{2} \int_P |P(f)| |V(f)| dS \\
= \sum_{n=0}^{N} \Pi_{A,n}(f, r)
\]

with the nth-order wave components

\[
\Pi_{A,n}(f, r) = \frac{|U|^2 (f) r^2}{2 \rho_0 c} \sum_{m=-n}^{n} |C_{nm}(f)|^2 \\
\left| h_n^{(2)}(kr) h_{n-1}^{(2)}(kr) - \frac{n+1}{kr} h_n^{(2)}(kr) \right|
\]

which neglects the phase relationship between particle velocity and sound pressure.

The critical distance \((r > r_{far})\) where the far field conditions are approximately are fulfilled can be calculated by

\[
10 \log \left( \frac{\Pi_A(f, r_{far}(f))}{\Pi(f)} \right) dB = 0.5 dB
\]
Radiation into far field

Radiation Pattern at 5kHz

SPL decrease by doubling the distance

Holographic directivity measurement

1/r law 6dB
Far field characteristics

Sensitivity

Sound Power

Directivity Index

Contour

Directivity Pattern at 5kHz

Balloon

Polar

Sensitivity

Sound Power

Directivity Index

Horizontal

Vertical

Holographic directivity measurement
Line Sources

Particularities:
- Large dimensions
- Multiple tweeter
- Wide spreaded near field ($r > > l$)

Problems:
- Sound field has high complexity
- Fitting for high frequencies (>5kHz) requires high order $N > 50$
- Many measurement points $M$, long measurement time

Rule of Thumb

$$N \approx \frac{l}{2} \cdot \frac{2\pi f}{c}$$

$$M > (N + 1)^2$$
Single Plane Symmetry (1PS)
symmetry axis aligned to the coordinate system $\phi_s = 0$

Simple coupling of the coefficients on the left side ($m < 0$) on the right side ($m > 0$)

$$C_{mn}(f) = C_{-mn}(f)(-1)^m \quad \text{with} \quad 0 \leq m \leq n \leq N$$

Reduced Number of Coefficients:

$$J = \frac{(N + 1)(N + 2)}{2}$$

Evaluating the single plane symmetry (1PS) by the metric

$$S_{1PS} = 1 - \frac{\sum_{n=1}^{N} \sum_{m=1}^{n} \left| (-1)^m (f)C_{-mn} - C_{mn} \right|^2}{\sum_{n=0}^{N} \sum_{m=-n}^{n} C_{mn}^2}$$

and predefined limit value (e.g. $S_{1PS} > 0.95$)
Dual Plane Symmetry (2PS)

symmetry axes $\phi_s = 0$ and $\phi_s = 90^\circ$ aligned to the coordinate system

Simple coupling of the coefficients on the left side ($m < 0$) on the right side ($m > 0$)

\[
\begin{align*}
C_{-(m-1)n}(f) &= 0 \\
C_{(m-1)n}(f) &= 0 \\
C_{mn}(f) &= C_{-mn}(f)(-1)^m
\end{align*}
\]

$m = 2s, s = 1,2,3$

Reduced Number of Coefficients:

\[
J = \begin{cases} 
\left( \frac{N}{2} + 1 \right)^2 & N = 0,2,4,... \\
\left( \frac{N}{2} + 1 \right)^2 + \frac{1}{4} & N = 1,3,5,... 
\end{cases}
\]

Evaluating the dual plane symmetry (2PS) by the metric

\[
S_{2PS} = 1 - \frac{\sum_{n=2}^{N} \sum_{s=1}^{n/2} (-1)^{2s} C_{2s,n} - C_{2s,n}^2 + \sum_{n=1}^{N} \sum_{s=0}^{n/2} C_{2s+1,n}^2}{\sum_{n=0}^{N} \sum_{m=-n}^{n} C_{mn}^2}
\]

and predefined limit value (e.g. $S_{2PS} > 0.95$)
Rotational Symmetry (RS)
no phi dependency

Condition for used Spherical harmonics:

\[ C_{mn} = 0 \quad m \neq 0 \]

Reduced Number of Coefficients:

\[ J = N + 1 \]

Evaluating the rotational symmetry (RS) by the metric

\[ S_{RS} = 1 - \frac{\sum_{n=1}^{N} \sum_{s=1}^{N} |C_{sn}|^2}{\sum_{n=0}^{N} \sum_{m=-n}^{n} C_{mn}^2} \]

and predefined limit value (e.g. \( S_{RS} > 0.95 \))
Reduction of Scanning Effort (System)

Example: wave expansion with maximum order N=30

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Number of Coefficients</th>
<th>Reduction of measurement samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Symmetry</td>
<td>961</td>
<td>0 %</td>
</tr>
<tr>
<td>Single plane symmetry</td>
<td>496</td>
<td>48 %</td>
</tr>
<tr>
<td>Dual plane symmetry</td>
<td>256</td>
<td>73 %</td>
</tr>
<tr>
<td>Rotational symmetry</td>
<td>31</td>
<td>97 %</td>
</tr>
</tbody>
</table>

Knowing the **symmetry properties** (a priori user input or automatic detection) can reduce the number of **measurement points** significantly.
Line Source

individual measurement of transducers

1) Measure each loudspeaker separately by using a multiplexer

2) Wave expansion of each loudspeaker

3) Super positioning of the multipoles

Benefits

- Directivity of individual transducers is less complex
- Automatic measurement, accurate positioning
- Accurate phase data
- Sound pressure at any point

Holographic directivity measurement
2 Multiplexing of Transducers

Simple Example: 2 Way Loudspeaker
Measurement Results

Superposition of individual measurements

2kHz
User defined Listening Zone

Step 1: Define a target listening area

Step 2: Extract representative curves

Summary Window collects most significant curves

e.g. spatial average + deviation of sound pressure level
IEC 62777 Standard using Listening Zone

Application: Personal audio devices, Laptops, Tablets, etc.
CEA2034 Standard

using Listening Zone

Application: Home audio devices, HiFi-Loudspeaker

Holographic directivity measurement
Conclusion

Measurement Targets:

- Directional characteristics
- Including boundary effects from the cabinet
- Far field (Pro Audio Line Arrays)
- Near Field (e.g. sound bars, studio monitor)
- Accurate Phase information
- Reasonable Time
Conclusion

Holographic measurement of line sources

- Comprehensive assessment of direct sound in 3D space (near + far field)
- High signal to noise ratio
- Suppression of room reflections (simulated far field conditions)
- Minimal influence air properties (air convection, temperature field)
- Automatic measurement minimizing positioning errors
- Low redundancy in the generated data set
- Directivity individual transducer by multiplexing or interleaved sweeping
- Accurate and comprehensive data set for simulation
- Spatial resolution can be controlled by order $N(f)$ of the expansion
- Spatial interpolation is based on acoustical model
Thank you!