Speaker Auralization – Subjective Evaluation of Nonlinear Distortion

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ABSTRACT
A new auralization technique is presented for the objective and subjective assessment of drivers in the large signal domain. Using the results of the large signal parameter identification a digital model of the particular driver is realized in a digital signal processor (DSP) to simulate the sound pressure output for any given input signal (test signal, music). This technique combines objective analysis and subjective listening test to assess the linear and distortion components in real time. This valuable tool shows the impact of each distortion component on sound quality and allows driver optimization with respect to performance, size, weight and cost.

Introduction
Loudspeakers which have similar small signal characteristics may sound quite different at higher amplitudes. Thermal and nonlinear mechanisms determine the maximal output of the driver and cause signal distortion. Most common measurement techniques evaluate the performance at minor amplitudes. Assessing the driver in the large signal domain says more about the performance of the speaker under normal working conditions. Loudspeaker are expected to be able to reproduce the sound as loudly as possible with low distortion. Cost, sensitivity, size and weight are other constraints which are directly related to the large signal performance. There are different objective approaches to measure the performance objectively. Harmonic and intermodulation distortion measurement show the effect of the nonlinearities for a special excitation signal. Nonlinear and thermal parameters show the physical causes of the distortion and are crucial for the driver design. However, the results of both measurements do not reveal the impact of the distortion on subjectively perceived sound quality. There are many other questions in the gap between subjective and objective assessment:

- How does the driver sound after optimization?

In order to give answers to this question we will present a new auralization technique which combines objective distortion measurements on audio signals in real time with a tool for performing systematic listening tests in the large signal domain. After giving a summary on large signal modeling we will apply this technique to artificial and natural sounds and will discuss the relationship between objective parameters and subjective sensations.

Loudspeaker Modeling

At low frequencies the driver may be modeled by a lumped parameter model comprising electrical and mechanical elements and state variables:

- \( u \) voltage at terminal
- \( i \) electrical input current
- \( x \) voice coil displacement
- \( v \) voice coil velocity (\( \frac{dx}{dt} \))
- \( F_m(x) \) reluctance force
- \( T_v \) voice coil temperature
- \( L_v \) voice coil inductance
- \( B_l(x) \) electrodynamic force factor (\( B_l \)-product)
- \( K_m(x) \) mechanical stiffness of driver suspension
- \( M_{ms} \) moving mass including air load
- \( R_{ms} \) resistance representing mechanical and acoustical losses

To model the dominant nonlinearities of the driver at high amplitudes this model considers the variation of the force factor \( B_l(x) \), the stiffness \( K_m(x) \), and the inductance \( L_v(x) \). Additional parameters \( L_2(x) \) and \( R_2(x) \) used for modeling para-inductance at higher frequencies are omitted here in this paper to keep the following equation as simple as possible.

The electromechanical equivalent circuit corresponds with the following set of nonlinear differential equations:

\[
\begin{align*}
  u &= R_{ms} + Bl(x) \frac{dx}{dt} + \frac{d(L_v(x)i)}{dt} \\
  Bl(x) + \frac{dL_v(x)}{dx} i &= M_{ms} \frac{d^2x}{dt^2} + R_{ms} \frac{dx}{dt} + K_m(x)x
\end{align*}
\]
Figures 2 shows the $Bl$-product of a driver measured by a dynamic identification technique (Distortion Analyzer [1]) and used as an example in this paper. Due to the short voice coil overhang the curve has a distinct maximum at 1.2 mm and decreases rapidly for positive and negative displacement.

![Bl-product versus displacement x](image1)

**Fig. 2.** $Bl$-product versus displacement $x$

The $Bl$-product is a very important motor parameter. It determines not only the driving force for a given current but also the electrical damping of the driver connected to an amplifier with low impedance output (normal voltage source). Thus, variation of the $Bl$-product versus displacement will produce two nonlinear terms in the differential equation. The two terms produce two different effects in the output signal. The first term $Bl(x)i$ may be interpreted as parametric excitation of the mechanical system where a function of $x$ is multiplied with the current $i$. The second term $Bl'(x)i$ in the nonlinear differential equation can be interpreted as nonlinear damping because a nonlinear function of $x$ is multiplied with the voice coil velocity. In both terms time signals are multiplied. This multiplication produces new spectral components in the output signal measured as harmonic and intermodulation distortion. The spectral properties of the two multiplied signals determine magnitude and spectral location of the distortion components. Since the displacement $x$ is a low-pass filtered signal a bass component of high amplitude is required to activate the nonlinear mechanism. The distortion components of the nonlinear damping have the highest amplitude around the resonance frequency where the velocity components have the maximal magnitudes. At higher frequencies the power spectrum of the distortion will decrease by 6 dB per octave. The spectral properties of the distortion generated by $Bl'(x)i$ (parametric excitation) depends on the current spectrum. If the inductance of the coil is a constant part which is higher than the alternating current $i$.

![Stiffness Kms(x) versus voice coil displacement x](image2)

**Fig. 3.** Stiffness $Kms(x)$ versus voice coil displacement $x$

The mechanical suspension of the example driver is represented by the stiffness $Kms(x)$ seen in figure 3. At higher amplitudes the stiffness increases rapidly indicating high mechanical load in the suspension. If $Kms(x)$ is not constant but varies with displacement the differential equation will have a nonlinear term $Kms(x)x$ which is the force at the suspension. Due to the multiplication of two displacement depending signals this term also produces nonlinear distortion components in the output signal. Since the power density spectrum of $x$ decreases by 12 dB per octave above the resonance frequency the magnitude of all distortion components falls rapidly above the resonance frequency. We expect that the nonlinear distortion caused by stiffness will only affect the bass reproduction.

![Voice coil inductance Ls(x) versus displacement](image3)

**Fig. 4.** Voice coil inductance $Ls(x)$ versus displacement $x$

The strong asymmetry of the inductance nonlinearity $Ls(x)$ shown in figure 4 is typical for drivers without short cut ring and pole cap. If the coil moves between pole plate and back plate (coil in) the inductance will be much higher than in opposite direction (coil out) where the magnetic field finds a much longer air path.

The nonlinear inductance yields two nonlinear effects: First, the variation the inductance introduces a nonlinear term $dLs(x)/dt$ in the differential equation which can be interpreted as a multiplication of a function of $x$ with current $i$ that is afterwards differentiated in the time domain. The multiplication of a function of $x$ with current $i$ is similar to the parametric excitation but the subsequent differentiation will enhance the distortion components by 6 dB per octave at higher frequencies.

The second effect of the varying inductance is the reluctance force $\int_i Ls(x)/dx$ which might be interpreted as an electromagnetic driving force. This term will vanish if $Ls(x)$ is constant and the derivative is zero. Due to the distinct asymmetry the derivative of $Ls(x)$ contains a constant part which is higher than the alternating current $i$.
Speaker Auralization - How it Works

In order to synthesize the loudspeaker output for any input signal in real time the set of nonlinear differential equations is transformed into the digital domain and implemented in a digital signal processor. Generating an identical copy of the driver in the digital domain makes it possible to check validity of the physical modeling, the nonlinear parameter measurement and the numerical calculations. Above that the digital modeling allows to modify the nonlinear transfer response of the driver virtually in order to investigate the effect of each nonlinearity systematically. There are two different ways for this modification:

In the simulation we would change the parameters of the driver and investigate the effect in the radiated output. The simulation is perfect for investigating design choices and to predict the performance of an improved design before the first prototype is finished. However, switching off the nonlinear parameters or any changes of the parameters will result in a different driver.

In the so called auralization we do not modify the properties of the driver investigated but enhance or remove the effect in the output signal. Thus the nonlinear differential equation and the parameters of the particular driver are kept unchanged during auralization. In the equation produces internal state variables (displacement, velocity, temperature) of the real driver. The output of the digital model is the radiated sound pressure signal \( p(t) \) in the far field of. However, in the nonlinear differential equation the sound pressure \( p(t) \) is the sum of a undistorted output \( p_{\text{out}}(t) \) and the distortion components \( p_u(t) \) and \( p_d(t) \). The undistorted is linearly related to the drivers input \( u(t) \) and the distortion components \( p_d(t) \) and \( p_d(t) \) are generated by nonlinear subsystems representing the nonlinear stiffness, BI-product and inductance, respectively (figure 5). The nonlinear subsystems are provided with the output \( p_{\text{out}}(t) \) forming a feedback loop, where the generated distortion components react to the state variables and their own generation process. This feedback loop explains the complicated behavior of the nonlinear system at large signals (compression, jumping effects). The Volterra series expansion, which corresponds to a feed-forward model, can not describe these mechanics precisely if the magnitude of the distortion components is of similar order as the magnitude of the linear input.

In the digital model used for auralization we copy the summing point in the feedback loop by tapping the linear signal \( p_{\text{out}}(t) \), \( p_u(t) \), \( p_d(t) \), \( p_d(t) \) and scaling them by the attenuators \( S_{\text{lin}}, S_{\text{K}}, S_{\text{Bl}}, S_{\text{L}} \) before summing up to the auralization output \( p(t) \). Changing the gain of the attenuators between 0 and 1 any desired ratio between the distortion components and the linear signal can be realized in order to determine the audibility of the distortion in listening tests. Clearly, setting all gain controllers equal one will yield the real driver output.

This topology of the driver model allows not only to adjust the magnitude of the distortion output. It allows also to measure the magnitude of the distortion components on-line while reproducing an audio-like signal. For such an input signal the distortion signals have a relative high peak value while the rms-value is much lower. Due to nonlinearity the crest factor in distortion components is much higher than in the original input signal \( u(t) \) or in the displacement signal \( x(t) \). Setting the peak value of the distortion in ratio to the peak value of the total output signal \( p_{\text{tot}} \) is an appropriate measure for the contamination of the reproduced signal.

If the driver under test is intended for a woofer or subwoofer channel an additional tweeter path can be realized by using a crossover as shown in figure 6. The input signal is attenuated by the gain controller \( S_{\text{w}} \) to produce displacement \( x \) that is typical for the application of the driver. The gain controller \( S_{\text{w}} \) applied to the output signal controls the volume of the sound at the headphone.

Performing Listening Tests

The Auralization technique allows performing systematic listening tests on loudspeakers to determine the audibility of the nonlinear distortion under normal conditions. These tests may be performed as blind or even double blind tests to avoid any bias and to check the reproducibility of the results.

First we will investigate the influence of the program material on the audibility of speaker distortion. Although, auralization technique makes it possible for the first time to measure the threshold of audibility in music, speech or other natural signals, the artificial test signals are also very interesting stimuli to become familiar with the character of the distortion and to use critical program material for which our ear is very sensitive. In this paper we start with a single excitation tone to test the audibility of harmonic distortion. In a second step we use a two-tone signal to check the audibility of intermodulation distortion.

Single Tone Stimulus

In our first experiment we supply a tone of 70 Hz to a driver with the nonlinear parameters given in figures 2,3 and 4. The amplitude was chosen to a voice coil displacement of \( x = 9.5 \text{ mm} \) typical for the particular driver.

The steady-state spectrum of the radiated sound pressure signal is shown in figure 7.
The component with the largest magnitude was produced by the fundamental tone. The large third-order and fifth-order harmonic components were caused by the symmetrical characteristic of the dominant nonlinearities. The fact that the second-order and the other even-order harmonics are more than 10 dB lower shows that asymmetrical parameter variations are much smaller than the symmetrical ones for the particular driver. Although, the magnitude of the distortion decreases with the order of the component, the ninth harmonic is only 25 dB below the fundamental. For an amplifier the high magnitude of the high-order harmonics would indicate a hard limiting nonlinearity (represented by high-order coefficients in a power series expansion).

Loudspeaker possess soft nonlinearities instead such as a linear or quadratic characteristics, which generates low-order harmonics in the first instance. The low-order harmonics are fed back to the nonlinearity again and again and thus are transformed into higher-order harmonics.

To investigate the effect of the Bl(x)-nonlinearity the gain controllers SK and SL are muted as shown in figure 8.

The spectrum of the auralized sound pressure signal p(t)' consists of the linear signal p_{lin}(t) and the distortion p_{Bl}(t) all nonlinearities are considered correctly in the calculation of the state variables.

At last Fig. 11 shows the spectrum of the distorted output signal of the digital model where the gain controllers SK and SL for Bl(x)- and Kms(x)-distortion are muted. Due to the distinct asymmetry in the Le(x)-curve the second-order and the other even-order harmonics are dominant.

After comparing the spectra we summarize that a single excitation tone does not reveal characteristic differences between Bl(x), Le(x) and Kms(x)-nonlinearities which make it possible to identify the cause of distortion and to determine the nonlinear parameter characteristic. Listening tests show only small differences in the subjective impression. Most of the listeners report that the additional harmonics are perceived as spectral enrichment of the tone. For an excitation tone at very low frequencies (below 40 Hz) close to our hearing threshold, the harmonic may even enhance the bass sensation which is preferred in some cases.
Although, the harmonics of a single tone does not sound unpleasant at all, the human ear is very sensitive in detecting the additional components. Listening test confirmed a threshold of audibility about 1-3 percent distortion. That corresponds to the results of psycho-acoustical modeling of our hearing where the distortion components just disappear in the masking level generated by the fundamental tone.

Multi-tone Stimulus

A single excitation tone is a very exceptional program material for normal loudspeaker applications. Even in woofers and subwoofers an audio signal has a certain bandwidth containing components of different frequencies. Exciting a driver with more than one tone additional components at the summed and difference frequency are generated. These intermodulation components do not fit in the overtone structure of the fundamental components. Using a simple two-tone signal for excitation we may investigate the generation of intermodulations in greater detail. Choosing the first frequency $f_1 = 70$ Hz and the second tone $f_2 = 800$ Hz all of the distortion components are nicely spaced and can easily interpreted. Figure 12 shows the spectrum of the sound pressure signal considering the effect of all nonlinearities.

The two fundamental tones can easily identified by the maximal amplitude. The first tone $f_1$ represents low frequency tone (bass) producing substantial voice coil displacement and the second tone $f_2$ represents a voice or any other musical instrument. The harmonics of the first tone $f_1$, give us the same information as the harmonics measured for the single tone shown in Fig. 7. However, equally spaced around the second tone $f_2$ we find the difference tone intermodulation at $f_i = f_2 - f_1$ and the summed tone intermodulations $f_i = f_1 + f_2$ with $i = 1, 2, ...$. It is quite interesting to observe that the second- and third-order intermodulation are almost 15 % of the fundamental $f_1$ and in the same order of magnitude as the third-order harmonic at $3f_1$. The harmonics of the second tone starting at $2f_2 = 1.6$ kHz are more than 20 dB lower than the intermodulations. Performing a conventional harmonic distortion measurement we would see distortion of high magnitude (> 30 %) at low frequencies only ($f < f_1$). At higher frequencies the measured harmonics are usually low (1–3 percent) giving the impression to have a sufficiently linear driver in the pass band.

Which nonlinearity produces such large intermodulations? Using the auralization technique with muted gain controller $S_R$ and $S_S$ we can see the contribution of the nonlinear suspension in figure 13. The intermodulations around $f_2$ are 20 dB smaller than the intermodulations in the total output. This can be explained by the spectral properties of the displacement $x$ which is the only signal in the nonlinear term $K_{ns}(x)$ causing distortion mainly at low frequencies. Increasing the frequency of the second tone $f_2$, the magnitude of the intermodulations will fall by more than 12 dB per octave.

Finally, only the gains $S_S$ and $S_R$ are activated in the digital model and the measured sound pressure spectrum is shown in figure 15. The intermodulations around the second tone $f_2$ exceed the harmonics of $f_1$ by more than 20 dB. This is caused by the nonlinear term $dL(x)/dx$. Although the tone $f_1$ produces large displacement, the small current at the resonance ($f_1 = f_2$) results in low harmonics of $f_i$. However, the product between displacement $x$ at $f_1$ with the high current at $f_2$ and the subsequent differentiation produces very high intermodulations. This distortion may be reduced substantially by applying a short cut ring or other means for linearizing and reducing the electrical inductance [2]. Again the harmonic distortion measurement can not reveal this kind of distortion affecting signals in the middle of the audio band.

The audibility of distortion in multi-tone signals depends very much from the number of tones and their frequencies. A sparse tone complex with a few tones is in most cases more critical than a dense spectrum because some of the distortion components will be masked by the excitation threshold due to neighboring fundamental tones. Fundamental tones and distortion components falling in one critical band width cause a roughness of the sound which is quite audible. Systematic listening test with two-tone signals lead to
almost the same thresholds of audibility as measured for single tones (1–3 percent). However, listeners can easily detect the non-harmonic structure and the roughness of the distorted sound which is perceived neither as natural nor as a pleasant effect.

**Dependence on Signal Amplitude**

There are at least two interesting properties of the nonlinear system which we do not find in a linear system. First, additional spectral components are generated in the output and second, the output signal also depends on the amplitude of the input signal. If we would measure a pure quadratic system (such as a squarer or an electromagnetic speaker manufactured 50 years ago using the reluctance force for excitation) the second-order harmonic and intermodulation distortion in the output would rise by 12 dB for a 6dB input increase. Drivers with a nonlinearity as shown in figures 2 - 4 have a much more complicated relationship between input and output amplitude. Since the nonlinearities are part of a feedback loop in the differential equation we also find some compression effects. In this part of the paper we will investigate the influence of the input signal amplitude. The amplitude is attenuated by 12 dB in order to get a maximal peak displacement \( x_{\text{peak}} \) which is the fourth part of \( x_{\text{max}} \). Figure 16 shows the spectrum of the output signal considering all driver nonlinearities. The second-order intermodulation distortion still exceed 10 % and they are more than 20 dB higher than the harmonics of \( f_1 \) and \( f_2 \).

**Tests with Audio Signals**

The auralization technique makes it very simple to perform systematic listening tests on a particular driver by using any audio signal. Due to the digital processing in the DSP the listener may change the working range of the driver and the linear and the nonlinear distortion components in real time. A vast amount of music and other audio sources can be surveyed to find program material which is either critical for the perception or typical for the particular application. The monitored peak values of displacement and distortion components are measured and recorded during the listening tests and may be used for determining the thresholds of audibility. A less experienced listener may also perform a short training before starting with the listening test to learn the detection of the loudspeaker distortion in a complex sound. The gain controller \( S_{\text{ac}} \) allows attenuating the linear signal to measure the audibility of the loudspeaker distortion in a complex sound. The gain controller \( S_{\text{ac}} \) allows attenuating the linear signal to measure the thresholds of audibility even in the small signal domain where the distortion are hardly audible. That may be important for defining the optimal working range of speakers intended for high-quality reproduction.

**Audibility of Speaker Distortion**

The perception of speaker distortion under common listening conditions depends on a variety of factors. Although also the audibility of linear signal distortion caused by variations in the
amplitude and phase response depends on the spectral and temporal properties of audio signals, the abilities of the listener and of course on the properties of the driver. The results of the listening tests performed with a two-tone signal are quite valuable for explaining the complicated mechanisms. The first fundamental requirement for generating substantial nonlinear distortion is that the audio signal contains low frequency components of sufficient amplitude to activate the displacement varying nonlinearities. As we have found for the two-tone signal high frequency components will produce additional intermodulation which are easy to detect. A stimulus with a sparse spectrum where the spectral components are clearly separated (for example a flute) keeps spectral masking minimal. The short but high strokes of a bass drum produce usually high peaks in distortion which are less audible due to the temporal masking effects. Bass tones with almost a steady-state characteristic (organ bass) enhance the perception substantially. Especially the high frequency components of the stimulus should be a natural sound (vocal, classical instrument) familiar to the listener. The experience, training, and the expectation of the listener are subjective but not less important factors determining the audibility. Including listeners with and without training shows the variation of the thresholds. Due to the complexity the threshold of the dominant woofer distortion can not be expressed by a few numbers. In a critical scenario trained listener may detect distortion at a very low level (1-5 percent) if program material is used, which has similar properties as the sparse multi-tone complex. For music with more spectral and temporal complexity the thresholds increases above 10%. In some material containing percussion and synthesized sounds with high transients and dense spectra even a trained listener may not detect even 30 percent distortion peaks.

Listening Examples
A practical demo of the auralization technique applied to artificial test signals and a representative music example may be found on the website http://www.klippel.net/papers.

Conclusion
Nonlinear speaker auralization is a new technique for assessing the large signal performance of drivers by combining subjective and objective approaches. The loudspeaker model considering the dominant nonlinearities is implemented in a digital signal processor (DSP). This way the radiated sound pressure signal in the far field can be calculated in real time for any given input signal. The output signal is the sum of the linear system response and the distortion components separated for each speaker nonlinearity. Each signal component may be attenuated separately to investigate the impact on sound quality. The peak values of the driver state variables (displacement, temperature) and the distortion in the reproduced audio signal are measured in real-time and may be compared with subjective judgments. This model uses the results of nonlinear system identification (linear, thermal and nonlinear parameters) as parameter input.

The auralization technique allows non-destructive testing at the limits and beyond. This test shows the dominant source of distortion for the particular driver and is the basis for defining the permissible working range (maximal displacement $X_{max}$). The listening test may be organized as a blind or open AB-comparison between different modifications of the real driver in order to determine the threshold of audibility systematically. The auralization makes the effect of even small distortion audible where other forms of subjective testing are very time-consuming and give no reliable results. This information is valuable not only for high-quality speaker but for drivers with an optimal performance/cost ratio as well. The auralization technique opens the dialog between engineers and users.

References