TFA Time Frequency Analysis

Software of the KLIPPEL R&D and QC SYSTEM (Document Revision 1.2)

FEATURES
- Wavelet transform
- Filter bank
- Short-time Fourier transform (STFT)
- High resolution over-laced analysis
- 3D Display (time slices)
- Waterfall spectrum
- Spectrogram
- Group Delay
- Signal Characteristics (peak, bottom, rms, crest, etc.)

APPLICATION
- 3D defect analysis (Rub & Buzz)
- Detecting acoustical and mechanical resonators (room modes, rocking modes)
- Visualizing Harmonic and Intermodulation Distortion

DESCRIPTION
Time Frequency Analysis is a processing module that visualizes the characteristics of an audio signal over time and frequency.

Processing is based on three different methods (Wavelet Transform, Short Time Fourier Transform, Filter bank) and can be applied to any kind of time signal e.g. from a measurement operation or an external wav-file.

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1 Principle

The Time Frequency Analysis is a calculation technique that provides a detailed view on the behavior of an audio signal. The method analyzes energy density in both frequency and time simultaneously. Similar to a music sheet it visualizes which frequency comes at which time.

Based on three different methods, the Wavelet Transform (WT), the Short Time Fourier Transform (STFT), or a Bark scaled Filter Bank Transform (FBA), the module decomposes the input signal and visualizes the signal characteristics over frequency and time.

2 Calculation Methods

2.1 Wavelet Transform (WT)

The Wavelet Transform is an analysis technique that correlates a signal with specified basic functions, so called Wavelets. Depending on the frequency, the length of the wavelet varies to optimize the relation between time and frequency resolution for each frequency band.

\[
W^\psi_{x}(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \psi^\ast\left(\frac{t-b}{a}\right) dt
\]

- \(x(t)\): signal in time domain
- \(\psi^\ast\left(\frac{t-b}{a}\right)\): conjugate complex wavelet function
- \(a, b\): scaling parameters

Complex Gaussian Morlet Mother Wavelet – Time Domain [2] [3]

The analysis uses the complex Gaussian Morlet Mother Wavelet which is represented in time domain by

\[
\psi(t) = \frac{1}{\sqrt{\pi B}} e^{i\omega_0 t} e^{-\frac{t^2}{\pi B}}
\]

and frequency domain by
\[ \Psi(\omega) = e^{-\left(\omega - \omega_0\right)^2 B^2 / 4} \]

with

\[ B = \frac{4}{(\omega_0 BW)^2} \]

\( B \): Bandwidth Parameter
\( \omega_0 \): Wavelet Centre Frequency
\( BW \): Bandwidth in Octaves

| Energy of the wavelet transform [5] | The Energy density function is defined by: \[ E^\Psi_x(f, t) = |W^\Psi_x(f, t)|^2 \] |

### 2.2 Short Time Fourier Transform (STFT)

The Short time Fourier transform uses a window function that is shifted successively over a time signal. Calculating the Fourier Transform of each windowed section provides the spectral information at each time. Limited by the uncertainty relation the results of this method are a compromise between frequency resolution and time resolution. The Energy density function over time and frequency is defined by:

\[
E(t, f) = \left| \int_{-\infty}^{\infty} e^{-j2\pi ft} x(\tau) h(t - \tau) d\tau \right|^2
\]

### 2.3 Filter Bank (FBA)

Another method to separate the spectral components is set of FIR-filter, which provides high time resolution.

### 3 Parameters

#### 3.1 Input

**Select**

The Source of the wave data has to be specified. Further Parameters appear depending on selected source.

- **File**
  - Absolute or relative path to a wave file.
- **Directory**
  - Absolute or relative path to a directory containing wave files.
- **Clipboard**
  - Paste waveform curve from other dB-Lab operations.
- **Operation**
  - Import waveforms from other measurement.
- **Imported**
  - Select data which is already imported.

#### 3.2 Processing

**Analysis Method**

This parameter sets the time-frequency analysis method to calculate the results.

- Wavelet Transform
- Short Time Fourier Transform
- Filter Bank

**Range Settings**

Parameter for selecting the frequency and time range of the signal for the analysis

- Start Time
- End Time
- Frequency Minimum
- Frequency Maximum

**Wavelet Transform**

**Bandwidth**

This parameter defines the time- and frequency resolution ratio. E.g.: more Wavelets dividing one octave correspond to an increasing frequency resolution in return for a decreasing time resolution.
Short-Time Fourier Transform

<table>
<thead>
<tr>
<th>Window Type</th>
<th>Window function which is used for the STFT (e.g. Hann, Rectangular, Tukey)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window Time</td>
<td>Length of the time windows</td>
</tr>
<tr>
<td>Window Overlap</td>
<td>Overlapping of the windows in percent</td>
</tr>
</tbody>
</table>

Filter Bank

| Number of Filters | Number of filter bands used for the analysis |

3.3 Display

<table>
<thead>
<tr>
<th>Normalization</th>
<th>None</th>
<th>No normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>To 0 dB</td>
<td>Maximum is set to 0 dB</td>
</tr>
<tr>
<td></td>
<td>To Peak Time Value</td>
<td>Each frequency band is normalized to its maximum</td>
</tr>
<tr>
<td></td>
<td>To fundamental</td>
<td>Result is normalized to t = 0 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPL Range</th>
<th>Displayed SPL range in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result Max</td>
<td>Maximum displayed SPL value in dB</td>
</tr>
<tr>
<td>3D: Number of Slices</td>
<td>Visualized time slices</td>
</tr>
<tr>
<td>Spectrogram: Colormap</td>
<td>Selection of the colormap of the spectrogram plot</td>
</tr>
<tr>
<td>Spectrogram: Number of Colors</td>
<td>Definition of the color step size either low, mid, high or fixed in dB</td>
</tr>
<tr>
<td>Spectrogram: Highlight Max Value</td>
<td>Checkbox to highlight the maximum value of the spectrogram plot</td>
</tr>
</tbody>
</table>

4 Results

<table>
<thead>
<tr>
<th>Waveform(t)</th>
<th>Time Domain</th>
<th>The graph shows the imported waveform. The time cursors are corresponding to the settings in the property page. You may move them with the mouse as well.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Waveform(f)</th>
<th>Frequency Domain</th>
<th>The Fourier Transform of the imported waveform. The frequency cursors for ( f_{\text{max}} ) and ( f_{\text{min}} ) are corresponding to the settings in the property page. You may move them with the mouse as well.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mother Wavelet (for Wavelet Transform only)</strong></td>
<td><strong>Time domain</strong></td>
<td><strong>Frequency domain</strong></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>----------------</td>
<td>---------------------</td>
</tr>
<tr>
<td></td>
<td><img src="image1" alt="Time domain" /></td>
<td><img src="image2" alt="Frequency domain" /></td>
</tr>
<tr>
<td>Real part $\Re(\psi(t))$</td>
<td>Imaginary part $\Im(\psi(t))$</td>
<td>Mother Wavelet $\Psi(\omega)$ + scaled Wavelets of one octave below</td>
</tr>
</tbody>
</table>

**Spectrogram Plot**

2D color representation of the Energy $E(t, f)$ as a function of time and Frequency.

**Waterfall Plot**

3D representation of the Energy Density.
**Signal Characteristics**

The TFA is calculating standard signal characteristics (e.g. mean, rms, peak, etc.) of the input waveform. This result window is showing these characteristics over time. In addition, the values of the signal characteristics are shown in the Fehler! Verweisquelle konnte nicht gefunden werden. window.

### Parameter

<table>
<thead>
<tr>
<th>y(t) Input Waveform</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3.49995</td>
<td>s</td>
</tr>
<tr>
<td>Channels</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Samples</td>
<td>154348</td>
<td></td>
</tr>
<tr>
<td>Sample Frequency</td>
<td>44.1</td>
<td>kHz</td>
</tr>
<tr>
<td>Bit depth</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

**Signal Characteristics (Full Signal)**

| Peak            | 0.64913 |
| Mean            | 0.00003 |
| Bottom          | -0.99997|
| RMS             | 0.18507 |
| Abs. Peak       | 0.99997 |
| Crest Factor    | 14.65298| dB    |
| Kurtosis        | 3.96700 |

**PDF Probability Density Function**

The graph shows the probability density function of the input signal.
5 Applications

Defect Analysis – Rub & Buzz

The picture shows the time frequency analysis of microphone signal of a chirp measurement. You can see that the chirp changes its frequency over time. At a fundamental frequency of 100Hz at t=0.7s, the loudspeaker excites a lot of high frequencies, which is a typical symptom for Rub and Buzz.

Waterfall spectrum - Room Response

The graph shows the waterfall spectrum of a loudspeaker measured in a normal office room. Below 200 Hz the plot shows distinct resonances with low damping which are caused by interaction with the room.

6 References


Find explanations for symbols at:  
http://www.klippel.de/know-how/literature.html

Last updated: June 04, 2021