Optimal Material Parameter Estimation by Fitting Finite Element Simulations to Loudspeaker Measurements

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Important characteristics for the sound quality of loudspeakers like frequency response and directivity are determined by the size, geometry and material parameters of the components interfacing the acoustic field. The higher-order modes after cone break-up play and important role in wideband transducers and require a careful design of the cone, surround and other soft parts to achieve the desired performance. Finite Element Analysis is a powerful simulation tool but requires accurate material parameters (complex Young's modulus as a function of frequency) to provide meaningful results. This paper addresses this problem and provides optimal material parameters by fitting the FEA model to an existing loudspeaker prototype measured by Laser vibrometry. This method validates the accuracy of the FEA simulation and gives further information to improve the modeling.

1 Introduction

Finite Element simulations become a powerful tool in the transducer design process only if they represent with sufficient accuracy the behaviour of the real loudspeakers. Since most of the information required to build a good simulation such as geometry and material densities, are available at the early stages of the design, the elasticity properties cannot be easily found at higher frequencies.

Even when the Young's modulus of the materials used in the assembly are measured based on standard techniques (e.g. ASTM E 756-93) using small sample taken from a flat sheet of the component and claimed as a hanging beam and excited pneumatically at the fundamental resonance as shown in Figure 1. The material parameters measured at low frequencies (< 150 Hz) used in Finite Element (FE) simulation do not describe the loudspeaker behaviour at higher frequencies. The soft materials (e.g. paper, rubber, fabric, plastic, …) used in the suspension and radiator shows a high dependency on the frequency and ambient temperature. Furthermore, the treatment of the components during the assembling process such as forming, gluing and combining to composite structures changes the material properties significantly.

In order to ensure accurate simulations for low and high frequencies, effective material parameters of the treated components as assembled in the final transducer need to be determined. Using the voice coil for the exciting the mechanical structure is also the best way to generate significant mechanical vibration at higher frequencies.

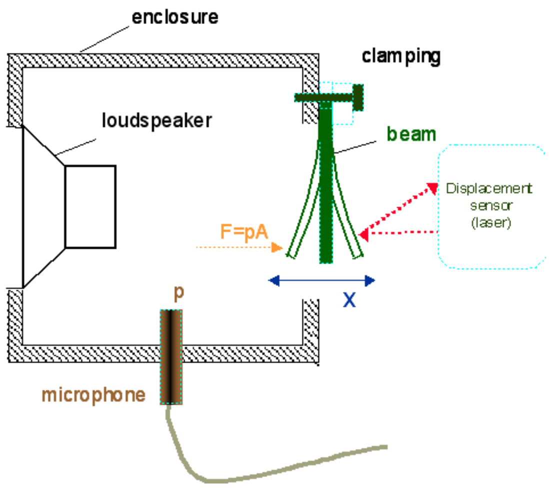

Figure 1. Simple beam technique to measure the Young's E-modulus and loss factor at low frequencies

Many techniques are developed for the updating of structural FE models in other disciplines like; bridges, helicopters [8], cars chassis, PCBs automotive etc. Some of these works [9] deal with the adjusting of geometrical or material parameters based on the minimization of the difference between the Frequency Response Functions FRF or the Modal parameters. Due to the nature of the material used on these structures, usually metals, concrete or hard plastics and the reduced frequency ranges of operation, the elastic properties in most of the cases can be modelled independent on the frequency and still producing accurate results.

In the case of the loudspeakers the engineers deal with devices which cover more than 10 octaves of bandwidth and a very diverse type of materials from soft like rubbers or foams, paper and other composites to hard ones like aluminium. Some of the materials used in the loudspeaker industry exhibit relevant viscoelastic characteristics and a FE simulation valid on the full band requires this frequency dependencies. This work deal with the estimation of the effective material parameters of the transducer components required for accurate simulations on the audio band using a laser scanner technique and Modal Analysis.

This paper proposes a technique to estimate the effective Complex Young's modulus of the materials scanned by laser vibrometer, which minimizes the error between the FE model and the scanned vibration of a real transducer. This fitting process is performed on the basis of modal parameters (Mode shapes and resonance frequencies) extracted from scanned vibration data and the eigenvalue solution (eigenvalues and eigenvectors) computed by the FE model.
Section 2 will describe the fitting process in greater detail followed by the presentation of the proposed Modal Analysis technique in Section 3 and updating of the FE model in section 4. In the last section the procedure will be illustrated on a practical example.

2 Optimal Parameter Estimation Procedure

The most convenient way to increase the agreement between the physical loudspeaker and its FE Model is to adjust the modal basis. The FE eigenvalue analysis is the most time and computationally effective way to calculate the modal parameters of the FEA model in contrast to the forced harmonic analysis, which requires the inversion of the FE matrices for the complete frequency range [8].

The general overview to extract the Complex Young's Modulus of a built transducer is shown in Figure 2.

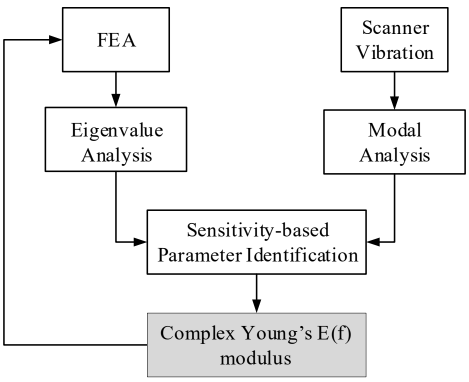

Figure 2. Overview of the material Parameter Estimation of loudspeaker components

The FE model provides the eigenvalues and eigenvectors of the structure corresponding to the natural frequencies, mode shapes and modal damping satisfying the homogeneous equation of motion with the boundary conditions. The vibration on the radiating surface is measured by laser vibrometry and used to extract the modal parameters of the physical transducer by Modal Analysis.
The simulated mode shapes and resonances frequencies (FEA eigenvalue solution) are compared with the measured mode shapes and resonance frequencies. An optimization algorithm based on the sensitivity of the FE eigenvalue solution with respect to the material changes of each component on the radiator surface, updates the parameters that reduces the difference between the mode shapes and resonance frequencies. This process is performed for all dominant modes found in the laser measurement producing the optimal material parameter at the different resonance frequencies.

3 Modal Analysis for Loudspeakers

3.1 Theory

The electromechanical transfer function (voltage displacement) H_x measured at each scanning point, is transformed into the pure mechanical transfer function $H_{x/F}$ via the *Bl* factor and the electrical impedance Z_e of the transducer at each frequency ω.

$$H_{X/F}(\omega) = H_X(\omega)\frac{Ze(\omega)}{Bl} \tag{1}$$

The modal analysis technique assumes that the vibration field measured on the transducer surface $X(\mathbf{r},\omega)$, can be represented by the superposition of an infinite amount of modes.

$$X(\mathbf{r},\omega) = \sum_{n=1}^{\infty} \boldsymbol{\varphi}_n(\mathbf{r}) q_n(\omega) \tag{2}$$

At each point $\mathbf{r}$ on the surface, the displacement is the product of the mode shape $\boldsymbol{\varphi}_n(\mathbf{r})$ and the modal resonator $q_n(\omega)$

$$q_n(\omega) = \frac{g_n}{\omega_n^2 - \omega^2 + j\eta_n\omega_n^2} \tag{3}$$

described by the resonance frequency ω_n, the modal damping factor η_n and the modal complex gain g_n.
In practice, only a few dominant modes have relevant impact on the mechano-acoustical performance of the driver and should be considered in the material parameter extraction process, simulation and in design optimization. The displacement produced by the superposition of these N dominant modes is called modal displacement X_{mod}:

$$X_{mod}(\mathbf{r},\omega) = \sum_{n=1}^{N} \boldsymbol{\varphi}_n(\mathbf{r}) q_n(\omega) \tag{4}$$

The difference between the measured and the modal displacements is called the residual displacement

$$X_{res}(\mathbf{r},\omega) = X - X_{\mathrm{mod}} \tag{5}$$

which contains the rest of low energy modes and the noise present in the measurement.

3.2 Modal Parameter Extraction

To extract the modal parameters of a vibration scanner measurement of the transducer, a technique based on Singular Value Decomposition (SVD) and circle fitting is developed see Figure 3.

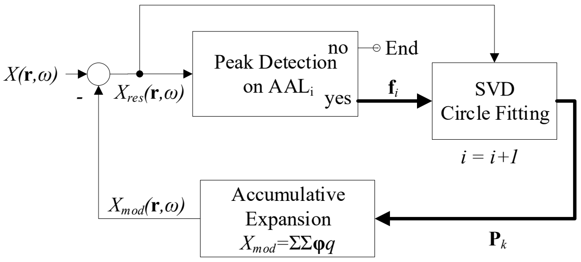

Figure 3. Processing loop to extract modal parameters from scanner measurements.

The measured displacement X($\mathbf{r}$,ω) is used to compute the Accumulated Acceleration Level (AAL_i) at the iteration i=1, which provides a measure of the mechanical energy integrated over the radiator surface[1]. An automatic peak detection routine provides the set of frequencies $\mathbf{f}_i$ corresponding with the most energetic modes present in the measurement, as shown in Figure 4. These frequencies are used to window the measured displacement transfer function X($\mathbf{r}$,ω) at frequencies close to the dominant resonance peaks.
Each of these w blocks of data $\mathbf{X}_w$ is arranged in a matrix containing the displacement of each measured point at each frequency of the window in its columns.
A Singular Value Decomposition of each block is performed in order to extract the dominant patterns found in the data:

$$\mathbf{X}_w = U\Sigma V^* = \sum_{k=1}^{K} \sigma_k \mathbf{u}_k \mathbf{v}_k^* \quad (6)$$

Where $\mathbf{u}_k$ and $\mathbf{v}_k$ are the left and right k-th singular vectors and σ_k is the k-th singular value of $\mathbf{X}_w$. Assuming that in the windowed block the participation of the dominant mode n is greater than the other contributing modes, the modal displacement $\mathbf{X}_{w,mod}$ in equation (4) comprising only the dominant mode n can be written in terms of the first singular values k=1 as follows:

$$\mathbf{X}_{w,\mathrm{mod}} = \boldsymbol{\varphi}_n(\mathbf{r}) q_n(\omega) \approx \sigma_1 \mathbf{u}_1 \mathbf{v}_1^* \quad (7)$$

Since the windows correspond to the resonance peaks where the particular modes are more excited, the first singular values of each SVD set provides the best representation of the shape and frequency response of the dominant mode, filtering out other orthogonal modes active in the region, other possible optical artefacts and noise produced in the measurement. Under this assumption, the following equivalences can be derived as:

$$\begin{aligned} \boldsymbol{\varphi}_n(\mathbf{r}) &= \mathbf{u}_1 \\ q_n(\omega) &\approx \sigma_1 \mathbf{v}_k^* \qquad \omega \in [\omega_l, \omega_u] \end{aligned} \quad (8)$$

The n-th mode shape $\boldsymbol{\varphi}_n$ can be extracted directly from the left singular vector $\mathbf{u}_1$ and the modal parameters of the resonator q_n(ω) need to be estimated from the product of the singular value σ_1 with the right singular vector $\mathbf{v}_1$ using a system identification method called circle fitting as illustrated in Figure 3.

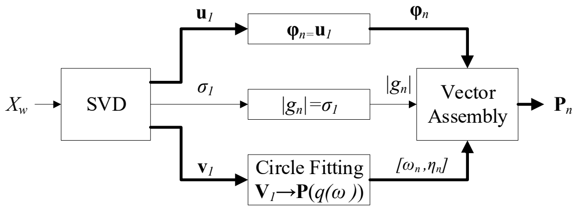

Figure 4. Internal process SVD and Circle Fitting for extraction of modal parameters

Even if the equivalence in (8) of the modal resonator and the singular value is valid only in the window interval [ω_l, ω_u], the estimated parameters are valid for the whole frequency band due to the assumption done in equation (2).
The full set of modal parameters $\mathbf{P}_n = \{\omega_n\ \eta_n\ g_n\ \boldsymbol{\varphi}_n\}$ of the dominant modes are extracted and the modal displacement can be computed using the accumulated superposition of the extracted modes according equation (4). Since each mode is extracted on a different window and no interaction between the modes is taken into account, a final fine tuning of all the modal parameters is required to guarantee that the superposition is consistent with the measured data.
The modal displacement X_{mod} is subtracted from the initial displacement generating a residual displacement X_{res} which is the input of the peak detection containing a new sets of lower energy modes that can be extracted in the same way. This process can be repeated until the residual vibration is only noise or vibration artefacts produced by the subtraction of non-orthogonal information contained in the previews modes.

3.2.1 Identification of Modal Resonators

Equation (4) provides a structure of the model to be identified using the scanner data, a set of N parallel poles without zeros. To keep the structure of the model, a Single Degree of Freedom (SDOF) algorithm is preferred for the loudspeaker because it focuses on the best fitting of the dominant peaks avoiding the undesirable effects of spurious poles, zeros and mathematical artifacts generated by complex structures used in the Multiple Degree of Freedom MDOF estimators like PolyMAX or other Least Squares algorithms [5].

To extract resonator parameters ω_n , η_n and g_n of equation (3), it is necessary to add two extra auxiliary terms to the model ΔK_n and ΔM_n describing the effects of lower and upper neighboring modes acting on the window [2]:

$$q_n^{SVD}(\omega, \mathbf{P}_v) = q_n(\omega) + \frac{1}{\Delta K_n} - \frac{1}{\omega^2 \Delta M_n} \qquad (9)$$

The parameter vector $\mathbf{P}_v = \{\omega_n\ \eta_n\ g_n\ \Delta K_n\ \Delta M_n\}$ is estimated minimizing the difference $\mathbf{E}_q$ between the real and imaginary part of the singular vector $\mathbf{v}_1$ and the expanded resonator q_n^{SVD}:

$$\mathbf{E}_q = \begin{Bmatrix} \Re\{\mathbf{v}_1(\omega)\} \\ \Im\{\mathbf{v}_1(\omega)\} \end{Bmatrix} - \begin{Bmatrix} \Re\{q_n^{SVD}(\omega)\} \\ \Im\{q_n^{SVD}(\omega)\} \end{Bmatrix} \qquad (10)$$

After discarding the ΔK_n and ΔM_n parameters, the remaining parameters provide optimal estimates of the modal resonator $\mathbf{P}_q = \{\omega_n\ \eta_n\ g_n\}$. The error produced by discarding ΔK_n and ΔM_n is later compensated by the fine-tuning process which includes the effect of neighboring extracted modes.

3.2.2 Improving Modal Interaction

Since the resonator parameters are estimated within individual windows, in some cases, where the peak is highly damped or the participation of surrounding modes on the same window is strong, the estimated parameters deviate from the optimal values.

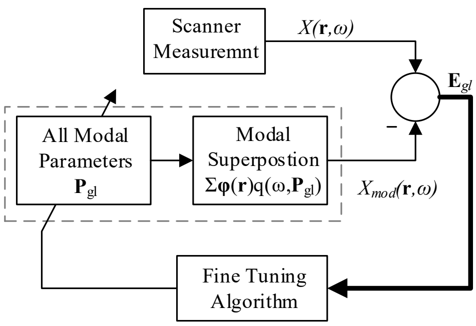

Figure 5. Fine tuning of modal parameters based on superposition of all modes

To cope with this problem a final refinement of all parameters of all the modes is done after completing the initial extraction process, reducing the global error E_{gl} between the measured displacement and the modal displacement X_{mod} composed by the superposition of the refined modes shown in Figure 5. This process guarantees a minimum fitting error between the N extracted modes and the measured displacement

$$E_{gl} = X(\mathbf{r}, \omega) - \sum_{n=1}^{N} \boldsymbol{\varphi}_n(\mathbf{r}) q_n(\omega, \mathbf{P}_{gl}) \qquad (11)$$

using the vector $\mathbf{P}_{gl} = \{\mathbf{P}_1\ \mathbf{P}_q\ \dots\ \mathbf{P}_N\}^T$ comprising the modal parameters of all N extracted resonators.
The global parameter vector $\mathbf{P}_{gl}$ is estimated by minimizing the error E_{gl} using a gradient descent algorithm. After convergence, the fined tuned modal parameters are the optimal parameters producing the best agreement between the measured data and the modal model.
The extracted mode shapes and fined tuned modal parameters are the target for the following FE simulation.

4 Parameter Updating

The detailed process to estimate the material parameters of the transducer components measured with a laser scanner on the surface is summarized in Figure 6.

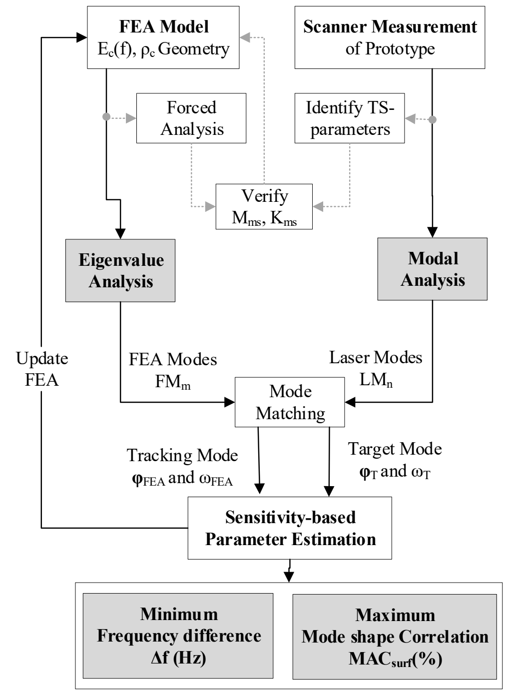

Figure 6. Process of model updating for the parameter estimation of the scanned components

The FEA model is built with the accurate geometry available at the design stage, the initial material parameters Young modulus $E_c(f)$ and densities ρ_c of the transducer components. The components of the sound radiating surface which are accessible by an Laser sensor are numbered with sub index c = {1 2 .. N_c}.
Each of the Laser Modes (Target Mode $\boldsymbol{\varphi}_T$) is selected one by one and the eigenvalue analysis is performed by searching the eigenvalues which absolute value is the closest to the target resonance frequency ω_T. A matching routine detects, from the computed set, the eigenmode exhibiting maximum correlation with the measured mode shape (Tracking Mode) and uses this to modify the complex Young's modulus systematically, maximizing the mode shape

correlation using the Modal Assurance Criteria MAC_{surf} [7] and minimizing the difference between the target and tracking resonance frequencies Δf.

Once the maximum mode shape correlation is achieved and the frequency difference is sufficiently small, the next higher-order mode can be updated following the same procedure. When all dominant target modes are optimized, the complex Young's modulus estimated at each resonance frequency represents the frequency dependent properties of the material.

4.1 FE Simulation of the Piston Mode

The full band response of the transducer including the piston and the higher order modes requires a full solution of equation for each frequency ω

$$\left(-\omega^2\mathbf{M} + i\omega\mathbf{R}(E_c(\omega)) + \mathbf{K}(E_c(\omega))\right)\mathbf{X} = \mathbf{F}(\omega) \qquad (12)$$

The matrices **M**, **R** and **K** comprise the mass, frequency dependent damping and stiffness of the finite element model respectively. The state vector **X** denotes of the displacements and rotations of the mesh nodes and the force vector **F** is the mechanical force exerted by the voice coil on the structure. The FE analysis with forced excitation can be computationally expensive and time consuming for complex models.

This method is used to verify the accuracy of the densities and the initial elasticity properties of the suspension parts (spider, surround), it is convenient to compare the mechanical Thiele-Small parameters of the FE model with the values measured on the real prototype. If the densities in the model are realistic, there should be a good agreement in the moving mass M_{ms}. An agreement between the measured and simulated values of the mechanical stiffness K_{ms} piston indicates that the geometry and the initial Young's modulus of the suspension parts are correct. Note that some differences can be found if the measurement is done in free air and no acoustic load is modeled with Finite Elements.

4.2 FE Simulation of Higher-Order Modes

Once the piston mode is correctly fitted, the mode shapes, natural frequencies and the damping of the higher-order modes in the FE simulation are calculated by Eigenvalue Analysis. This method solves the homogenous equation

$$\left(\mathbf{M}^{-1}\mathbf{K}(E_c(\omega)) - \omega_m^2\mathbf{I}\right)\mathbf{\Phi}_m = \mathbf{0} \qquad (13)$$

where $\mathbf{\Phi}_m$ are the m mode shapes [10] and ω_m are the complex eigenvalues corresponding to the squared resonance frequency. The real part of ω_m is the natural frequency while the ratio between the imaginary and the real parts of the eigenvalues represents the modal loss factor. This analysis is much faster than solving the full dynamic problem with the forced excitation **F** on the right-hand side of equation . This is very convenient for the material parameter estimation where several iterations and gradient computations of the model are required.

Since the stiffness matrix in equation depends on the frequency, this constitutes nonlinear eigenproblem requiring a nonlinear eigenvalue solver [11] and [12] not easily available in most commercial FEA packages. A practical way to reduce the complexity of the problem is to solve searching for the eigenvalues closest to the target resonance frequencies ω_T, extracted from the scanned prototype. Assuming that the material parameter $E_c(\omega)$ of the model is constant around the target frequency:

$$\begin{gathered} E_c(\omega_T) \rightarrow E_c \\ \left(\mathbf{M}^{-1}\mathbf{K}(E_c) - \omega_m^2\mathbf{I}\right)\mathbf{\Phi}_m = \mathbf{0} \end{gathered} \qquad (14)$$

the stiffness matrix **K** depends on a constant value, the eigenvalue problem becomes linear and the mode shapes and resonance frequencies correspond to the material parameter value of each component E_c given at the particular target frequency ω_T.

4.2.1 Mode Matching

From the set of the m computed modes, only the most correlated with the current target mode shape is retained and used to update the material parameters. This is the selected FEA mode $\mathbf{\Phi}_{FEA}$ with resonance frequency ω_{FEA}.

If the material parameters E_c of the model tends to the effective value of the physical prototype $E_{c,eff}$, the FE mode shape $\mathbf{\Phi}_{FEA}$ and resonance frequencies ω_{FEA} get closer to the target mode shapes $\boldsymbol{\varphi}_T$ and resonance frequencies ω_T.

$$E_c \rightarrow E_{c,eff} \Rightarrow \begin{cases} \mathbf{\Phi}_{FEA} \rightarrow \boldsymbol{\varphi}_T \\ \omega_{FEA} \rightarrow \omega_T \end{cases} \qquad (15)$$

The idea is to find the effective material parameters of the prototype reducing the error between the simulated and measured modal basis, this process is detailed in the next section.

4.2.2 Estimating Effective Parameters

In the proposed approach, the parameter estimation of the updating components is achieved by means of the sensitivity method [12] based on the linearization of the generally non-linear relationship between the measurable outputs of the model and the input material parameters of the components E_c.

The difference $\boldsymbol{\varepsilon}$ between the outputs of the FEA model $\boldsymbol{\mu}_{FEA}=\{\omega_{FEA}\ \mathbf{\Phi}_{FEA}\}^T$ and the measured outputs

$\boldsymbol{\mu}_{EXP}=\{\omega_T \quad \boldsymbol{\Phi}_T\}^T$ of the scanner vibration is represented as a truncated Taylor expansion after the linear term

$$\boldsymbol{\varepsilon} = \boldsymbol{\mu}_{\mathrm{EXP}} - \boldsymbol{\mu}_{FEA}(\mathbf{E}) \approx \mathbf{r}_i - \mathbf{S}\left(\mathbf{E} - \mathbf{E}_i\right) \qquad (16)$$

Where $\mathbf{E} = \{E_1\ E_2\ \ldots\ E_{Nc}\}^T$ is the vector containing the material parameters of the updating components, $\mathbf{r}_i = \boldsymbol{\mu}_{EXP} - \boldsymbol{\mu}_{FEA}(\mathbf{E}_i)$ is the residual defined at the ith iteration and $\mathbf{S}$ is the sensitivity matrix

$$\mathbf{S}_i = \left[\frac{\partial \boldsymbol{\mu}_{FEA}}{\partial E_1}\right]_{\mathbf{E}=\mathbf{E}_i} = \begin{bmatrix} \dfrac{\partial \omega_{FEA}}{\partial E_1} & \cdots & \dfrac{\partial \omega_{FEA}}{\partial E_{Nc}} \\ \dfrac{\partial \boldsymbol{\Phi}_{FEA}}{\partial E_1} & \cdots & \dfrac{\partial \boldsymbol{\Phi}_{FEA}}{\partial E_{Nc}} \end{bmatrix}_{\mathbf{E}=\mathbf{E}_i} \qquad (17)$$

relating the changes of the model resonance frequency and mode shape with respect to each of the material parameters of the N_c components. Since the sensitivity matrix is computed at the current vector $\mathbf{E}=\mathbf{E}_i$ and the difference $\boldsymbol{\varepsilon}$ is assumed to be small for these parameters, the correction factors to be applied to the parameters $\mathbf{\Delta E}= \mathbf{E}-\mathbf{E}_i$ required to minimize residual in (16) can be computed with the pseudoinverse of the sensitivity matrix as

$$\Delta \mathbf{E}_i = (\mathbf{S}^T \mathbf{W} \mathbf{S})^{-1} \mathbf{S}^T \mathbf{W} \mathbf{r}_i \qquad (18)$$

Where the weighting matrix $\mathbf{W}$ is chosen accordingly to the deformation of the material component on the selected mode vibration. Then the material parameters for the next i+1 iteration are updated as

$$\mathbf{E}_{i+1} = \mathbf{E}_i + \Delta \mathbf{E}_i \qquad (19)$$

This process is repeated until the parameters converge to stable values.

5 Case Study

The proposed method is applied to midrange-woofer loudspeaker with three components to be updated.

5.1 DUT Description

The example loudspeaker is a 6-inch woofer which uses a dust cap made of plastic, paper cone covered on the back by a fiber increasing considerably the bending stiffness and the surround is made of black rubber as seen in Figure 7

Figure 7. Woofer prototype used for estimation of the dust cap, cone and surround material parameters

The first step in the process is to extract the modal parameters as presented in section 3.2.

5.2 Modal Parameters

A fast scanner measurement consisting of 50 points distributed along a radial line on the surface produces the AAL shown in Figure 8.

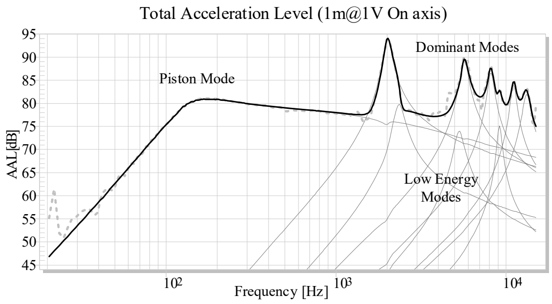

Figure 8. Modal analysis of the scanner measurement, (dashed grey line) measured AAL, (solid black line) AAL of modal expansion and (solid grey lines) modal resonators.

There are 9 modes 7 clearly dominant and 2 with lower energy seen on the AAL extracted as described in section 3.2. The modal resonators comprising the parameters in equation (3) fit accurately the peaks and the modal expansion (2) is the optimal superposition describing the measurement using the 9 modes with the refinement of the modal parameters described in section 3.2.2. Note that the modal analysis reduces the complexity of the vibration to the superposition of few modes relevant for the acoustic performance and containing valuable information of the elasticity properties of the materials of the scanned components deformed on the mode shape.

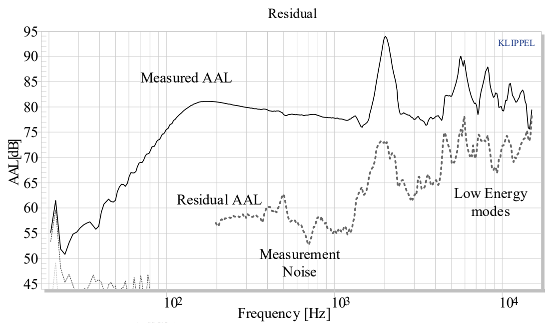

Figure 9. Residual AAL after subtracting the modal expansion containing 9 modes (solid line) Measured AAL, (dashed line) Residual AAL

The AAL of the residual displacement after subtracting the modal expansion including Figure 9 shows the existence of other low energy modes and the measurement noise. These modes can be extracted continuing the iteration process shown in Figure 3. In Table 2 the Modal parameters of the 9 extracted modes are detailed.

Mode	$f_n=2\pi\omega_n$ [Hz]	$Q_n=1/2\eta_n$	$\lvert g_n \rvert$ [dB]	Info
0	127.4	3.14	133.5	Piston
1	1999.6	11.5	132.4	1st Breakup

2	2346.2	7.18	120.3	Low energy
3	5283.4	8.07	117.6	Low energy
4	5671.5	11.16	131	2nd Breakup
5	8173.6	13.17	127.6	4th Breakup
6	9156.1	14.29	116.9	5th Breakup
7	11066.6	12.72	125.9	6th Breakup
8	13201.5	9.73	129.8	7th Breakup

Table 1. Modal parameters

In this example the piston and the first two breakup modes are selected as targets to update the FE model and estimate the Young's modulus of the dust-cap, cone and surround materials at these particular frequencies.

5.3 Updated Modes

The measured Target and the optimized FEA mode shapes of the three selected candidates for the optimization are shown in Figure 10.

The values of the mechanical parameters in Table 2 deviate less than 4% this indicates that the densities of the whole structure and the Young's modulus of the suspension parts at the piston mode frequency 127 Hz in the model are consistent with the built prototype.

Parameter	Measured	FEA	Difference [%]	Unit
M_{ms}	5.78	5.99	-3.6	g
K_{ms}	3.78	3.84	-1.6	N/mm
R_{ms}	1.53	1.53	3.9	Kg/s

Table 2. Mechanical Thiele-Small parameters

For this transducer the next updated mode is the first breakup at 1999 Hz corresponding with highest peak found in the measured AAL. After optimizing the FEA model by adjusting the Youngs modulus of the difference between the resonance frequencies is less than 1 Hz and the mode shape correlation MAC_{surf} is 98%. Note in the center plot of Figure 10 the good agreement between the measurement and the optimized mode.

Target Mode	$F_T=2\pi\omega_T$ [Hz]	$F_{FEA}=2\pi\omega_{FEA}$ [Hz]	Δf [Hz]	MAC_{surf} [%]
T_0	127	127.2	-0.2	100
T_1	1999.6	1999.8	-0.16	98
T_2	5671.5	5677.6	-6.11	87

Table 3. Summary of updated modes

The last updated mode is the third peak found in the AAL at 5671 Hz its mode shape can be seen in the last plot of Figure 10. Note that the measured mode shape has some noise due to the small amplitude displacement of the high frequencies, this lead a correlation of 87% on the surface and a resonance frequency difference of 6.1 Hz.

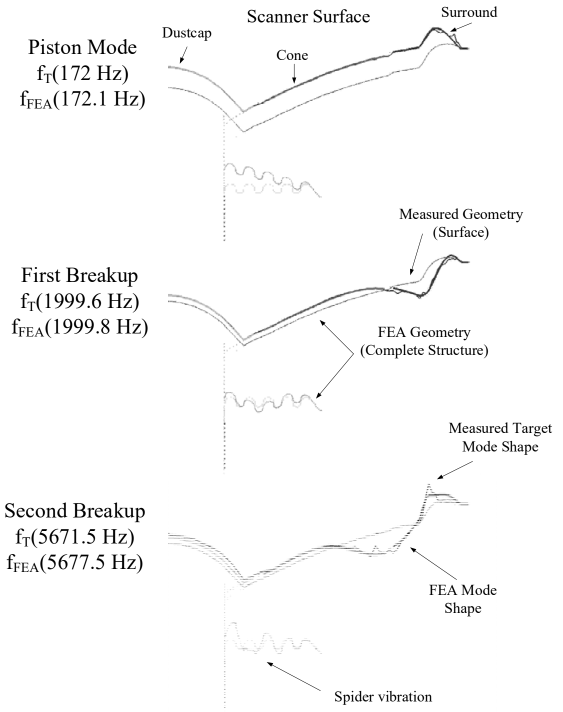

Figure 10. Superposition of the measured Target $\mathbf{\Phi}_T$ and the updated FEA $\mathbf{\Phi}_{FEA}$ mode shapes

From Figure 10 can be clearly seen that the estimated parameters produces the highest posible correlation on the surface which is the part of the mode shape that can be measured. Since the other componenst like the spider, voice coil and former are not scanned with a laser scanner, no Elasticity properties can be extracted usign the FEA model. The Young's modulus of the three components used in the optimization are presented in section 6 Results and Discussion.

5.4 Forced Response (Voice coil excitation)

The eigenvalue analysis is fast and convenient to examine the global characteristic of the vibration but a forced analysis including the voice coil excitation is required to simulate real life conditions and apply design changes.

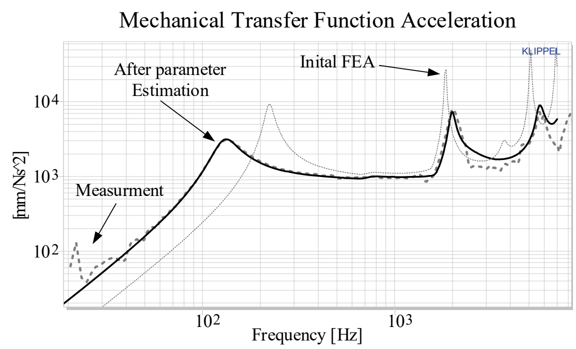

Figure 11. Mechanical response before and after Parameter Estimation.

Figure 11 presents the purely mechanical response integrated on the surface of the measurement and the FE simulation. The electrical effects are removed as described in equation (1) eliminating the electrical damping and the high frequency inductance attenuation.

The first simulation based on material parameters found in previews studies [14] presents a substantial difference with the measurement, the resonance peaks are shifted from the measured positions and the distance between adjacent resonances is too short. After updating the three modes, the model response fits more accurately the measurement specially on the resonance peaks which are placed at exact positions less than 6 Hz deviations and the distance between them agrees. The effective material parameters required to produce these results are frequency dependent and are described in the following section.

6 Results and Discussion

The optimal estimates of the material parameters obtained at the different target frequencies are linearly interpolated to generate values of the frequencies in between.

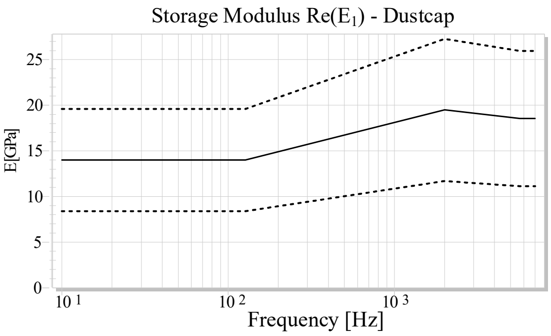

Figure 12. Young's modulus $E_1(f)$ of the dustcap. (solid line) modelled value, (dashed lines) range of values producing 5% change of resonance frequency

As seen from the mode shapes in Figure 10, the dustcap remains almost rigid on the full band used for the estimation. The real part of the Young's modulus depicted in Figure 12, note that the dashed lines corresponding to the range of values that the parameter can take providing only 5% of changes on the resonance frequencies are wide. This information is derived from the sensitivity matrix (17) and means that the Young's modulus of the dust cap cannot be accurately measured because the material is not deformed even at frequencies up to 5 kHz (third break-up mode). It is concluded that the dust-cap has no impact of the dynamic response of this driver.

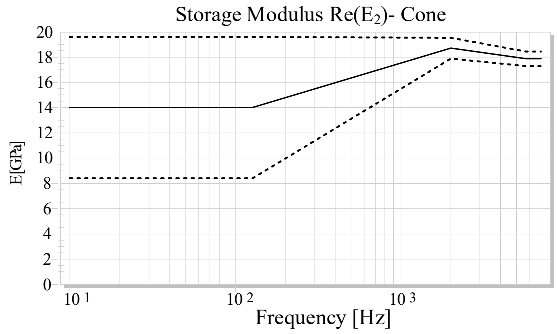

Figure 13. Young's modulus $E_2(f)$ of the Cone (solid line) modelled value, (dashed lines) range of values producing 5% change of resonance frequency

The cone material used on this example loudspeaker is a composition of formed paper with a high bending stiffness, it explains the large plateau area between the piston mode and the first breakup. Not from the lack of sensitivity (dashed lines) that at the low frequencies the parameter cannot be uniquely defined because the cone is not deformed. After the first breakup the cone interacts with the surround and becomes more sensitive Figure 10. Note that the material behaviour is almost constant with respect to the frequency with some variations at high frequencies.

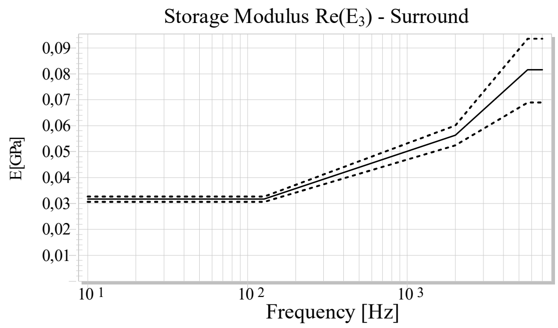

Figure 14. Young's modulus $E_3(f)$ of the Surround (solid line) modelled value, (dashed lines) range of values producing 5% change of resonance frequency

At the fundamental resonance of the driver (piston mode), the surround material is much more involved in the vibration and it makes the model very sensitive to it, note the dashed lies are very close to the parameter value. At high frequencies, the material gets harder as expected due to the viscoelastic characteristic of the rubber and its impact on the break modes reduces because the cone starts to dominate the vibration.

7 Conclusions

This paper proposed a methodology to estimate the elastic material parameters of the loudspeaker components based on the optimization of the modal basis the FE model providing more accurate simulations and optimal matching with the scanner measurements. The method exploits the fast-computational performance of the eigenvalue analysis to extract the Youngs modulus of different components at the resonance frequencies of the built prototype.

The optimization of the FE model is achieved by means of the inversion of the Sensitivity matrix which provides as well valuable information for the transducer design. For instance, the determination of the exact regions where each of the components participate or not on the global vibration provides the designer a better way to decide which components should be modified to tune determined mode. This has a deep implication on the design of full band loudspeakers which acoustical performance need to be optimized beyond the piston mode.

A novel method to extract the modal parameters of laser scanner measurements based on Singular Value Decomposition and System identification techniques is presented. This method preserves the parallel pole structure meaningful for mechanical analysis of transducers, overcoming the problems of other techniques requiring zeros and higher amount of artificial modes to fit the measured data.

The effectiveness of the method is shown with the case study of a 6-inch woofer which estimated effective parameters produce a good fitting on the full audio band.

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