

Active Compensation of Transducer Nonlinearities

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ABSTRACT

Nonlinearities inherent in electromechanical and electroacoustical transducers produce signal distortion and limit the maximal amplitude of the output signal. Assessing the large signal performance has been a subject of acoustical research for many years providing nonlinear models and new methods for the measurement of the large signal parameters. The identified model allows prediction and simulation of the nonlinear behavior and direct comparison with measured symptoms. The good agreement between model and reality is the basis for developing novel digital controllers dedicated to transducers that compensate actively for nonlinear distortion by inverse preprocessing of the electrical input signal. This paper gives a summary on the activities in the last 15 years and new challenges of the future.

1 Introduction

All loudspeakers operated at sufficiently high amplitudes produce signal distortion which are not generated in the small signal domain. This kind of distortion is a symptom of nonlinearities inherent in the loudspeaker. It impairs the perceived sound quality and indicates that the loudspeaker is operated close to the physical limits. If the distortion become audible the listener usually attenuates the signal to protect the loudspeaker against thermal or mechanical damage. In the past the large signal domain is not intended for normal operation but is more used as a safety range or head room. However the on-going trend to smaller loudspeakers giving more output at higher quality make it necessary to exploit any unused resources and to cope with nonlinear and thermal mechanisms in loudspeakers. The large signal modeling and the measurement of the loudspeaker parameters reveal the causes of the distortion and the relationship to the results of traditional distortion measurements. This information is crucial for the design of the driver and the passive loudspeaker system. However, a linear loudspeaker is not necessarily optimal. If the size, weight, efficiency and maximal output is also a point of concern, some of the loudspeaker nonlinearities are accepted as desired properties and are separated as regular nonlinearities from excessive nonlinearities caused by loudspeaker defects [42]. For example a systematic offset of the voice coil position or an asymmetric suspension produces excessive distortion and unstable behavior. This can be avoided or easily fixed by a proper design and consistent manufacturing. The regular loudspeaker nonlinearities are the main target of active linearization and other forms of electrical control.

This paper gives a summary on the activities in this field.

2 Glossary of Symbols

The following symbols are used within the paper:

$i(t)$	input current
$u(t)$	voltage at terminals
$x(t)$	voice coil displacement
$v(t)$	velocity of voice coil
$a(t)$	acceleration
$p(t)$	sound pressure
ΔT_v	increase of voice coil temperature
\mathbf{x}	state vector
\mathbf{z}	transformed state vector used in normal form
$q_a(t)$	volume velocity of the air in the port
$d(t)$	distortion generated in the loudspeaker model
$R_e(T_v)$	electrical voice coil resistance at DC depending on voice coil temperature
L_e	voice coil inductance at low frequencies
M_{ms}	mechanical mass of driver diaphragm assembly including air load and voice coil
R_{ms}	mechanical resistance of total-driver losses
$K_{ms}(x)$	mechanical stiffness of driver suspension
$C_{ms}(x) = 1 / K_{ms}(x)$	mechanical compliance of driver suspension
$Bl(x)$	force factor (Bl product)
B	magnetic induction
$F_m(x, i)$	reluctance force due to variation of $L_e(x)$

f_s	resonance frequency of the mechanical system
M_p	acoustic mass of air in the port,
C_B	acoustic compliance of air in enclosure,
R_p	acoustic resistance of port losses,
$s=j\omega$	Laplace operator,
$H_x(s)=X(s)/U(s)$	linear transfer function between voltage and displacement,
$H_c(s)$	linear system function of the controller,
$K(s)$	loop gain in servo control
$H_f(s)$	input filter in servo control
h_i	nonlinear subsystem in loudspeaker model
g_i	nonlinear subsystem in controller
$G_i(s)$	linear subsystem in controller
P	parameter vector
A	nonlinear states
τ	time delay

3 Basics of loudspeaker linearization

3.1 Nonlinearities in Loudspeakers

The loudspeaker may be considered as a single-input-multiple-output system (SIMO) as illustrated in Fig. 1. Using an amplifier with low impedance output the voltage $u(t)$ at the loudspeaker terminals may be considered as the single input while the sound pressure signal $p(t, \mathbf{r}_i)$ at the point \mathbf{r}_i in the sound field is one of the multiple outputs. The signal flow between input and output may be modeled by a chain of linear and nonlinear subsystem, illustrated by thin and bold blocks, respectively, in Fig. 1. The power amplifier, passive crossover and other electronic circuits (e.g. for protection) may be considered as linear at the amplitudes used in praxis. The following electro-mechanical transducer is nonlinear using a moving coil in a static magnetic field and a mechanical suspension system. The voice coil displacement $x(t)$ may be considered as the output of this subsystem and is also the state variable that is directly related with the nonlinearities inherent in this block. This voice coil current $i(t)$ at the electrical terminals reflects not only the state of the electrical parts but also the mechanical vibration.

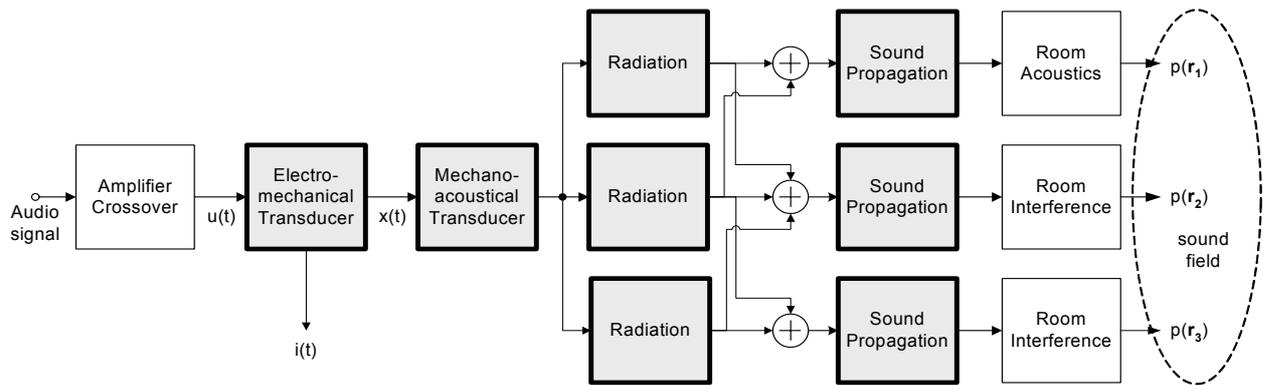


Fig. 1: General model describing the basic signal flow in loudspeakers using linear (thin) and nonlinear (bold) subsystems.

The mechano-acoustical transducer uses the diaphragm, the enclosure or a horn to transform the voice coil displacement into an air flow. The air in a sealed box or the air between diaphragm and phase-plug in a horn loaded compression driver behaves nonlinear if the variation of the air volume is not negligible.

In many loudspeakers the radiation of the sound has to be modeled by multiple subsystem which comprise different nonlinearities and disperse the signal in different direction. For example, a driver mounted in a vented enclosure or coupled with a passive radiator has two separate sound sources. The nonlinearities of the air flow in the port or the nonlinear suspension of the passive radiator will produce distortion which are not in the sound radiated from the diaphragm of the active radiator.

The Doppler effect depends on the radiation angle and produces phase modulation between low and high frequency components. The radiation in axis of the driver produces maximal Doppler distortion because the direction of radiation coincides with the displacement of the cone. A low frequency component also produces amplitude modulation of a high-frequency component if the cone moves with respect to the acoustical boundaries (frame, enclosure, baffle) and the radiation condition changes.

In horn compression drivers the propagation of the sound wave becomes nonlinear if the sound pressure is high. A pressure maximum travels faster than a pressure minimum causing a gradual steepening of the wave front.

The interaction with the room, the superposition of the direct sound with reflected waves may be modeled by a linear system because the air behaves sufficiently linear.

3.2 Constraints in loudspeaker linearization

The active linearization techniques use an electrical controller connected to the input of the loudspeaker. If the nonlinearities are located in the subsystems connected in series to the input and all of the remaining subsystems in the output branches behave linearly then the sound pressure in the complete sound field can be linearized. However, if the nonlinearity in the radiation and sound propagation depends on the path of the wave then the sound pressure can be linearized at one point only. Fortunately, most of the dominant nonlinearities in the motor, suspension and in a horn can be modeled by a single input and single output system and can be compensated before the signal is dispersed. The Doppler effect is an example where active compensation is limited to a certain radiation angle. Here nonlinear

control is limited in the same way as the equalization of the linear amplitude response.

4 Analogue linearization techniques

Since the loudspeaker is still an analogue device, a controller realized with analogue electronics is the first choice for active linearization.

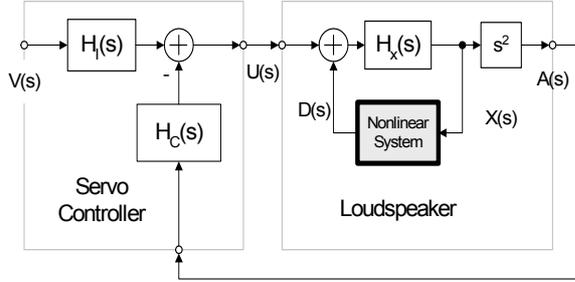


Fig. 2: Servo control of a loudspeaker by using feedback of voice coil acceleration

4.1 Servo control using output feedback

The first approach [1] - [7] to active loudspeaker linearization used negative feedback of the measured output signal as illustrated in Fig. 2. The loudspeaker is modeled by a linear path comprising the linear system $H_x(s)$ and second-order differentiator (represented by the squared Laplace operator s^2) transforming the voice coil displacement X into the acceleration A . A nonlinear system models the effect of the motor and suspension nonlinearities depending on the voice coil displacement. The output D may be interpreted as nonlinear distortion added to the input voltage U . The acceleration A of the cone may be measured by using an accelerometer and supplied to the servo controller. The controller comprises a first system $H_c(s)$ to realize a desired open loop gain

$$K(s) = H_c(s)H_x(s)s^2 \quad (1)$$

The magnitude of the transfer function

$$\frac{A(s)}{D(s)} = \frac{H_x(s)s^2}{1 + K(s)} \quad (2)$$

between the Laplace transformed distortion $D(s)$ and the acceleration signal $A(s)$ may be reduced by increasing the loop gain $|K(s)|$ in the denominator. Thus the distortion can be attenuated by servo control without modeling the nonlinear mechanisms in greater detail.

The linear system $H_1(s)$ at the controller input is used to realize a desired linear transfer function

$$\frac{A(s)}{V(s)} = \frac{H_1(s)H_x(s)s^2}{1 + K(s)} \quad (3)$$

between controller input $V(s)$ and acceleration $A(s)$. Using $H_1(s)=1+K(s)$ the controller preserves the original small signal behavior (resonance frequency and loss factor) but compensates for the loudspeaker nonlinearities only.

However, servo control applied to loudspeakers has drawbacks:

- The transfer function $H_c(s)$ of the controller has to be adjusted to the transfer function of the loudspeaker to ensure stability.
- A high-quality sensor is permanently required which monitors the displacement, velocity or voice coil acceleration. The measurement of the sound pressure is not practical for two reasons: First, a microphone even located in the near field of the driver will cause a small time delay and the corresponding phase shift in $K(s)$ requires an attenuation of the $K(s)$ at higher frequencies to preserve stability in the feedback. Second, ambient noise monitored by the microphone will be compensated at the microphone position but the amplified signal will also be radiated to other points in the sound field.
- A malfunction of the sensor or electronics in controller may also damage the loudspeaker.
- A protection system is required to attenuate the input signal when thermal or mechanical overload may damage the speaker.

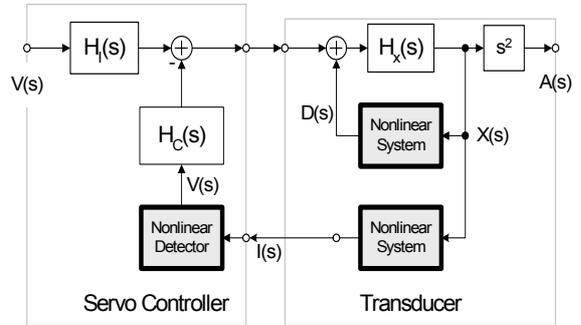


Fig. 3: Servo control of a loudspeaker by using measured voice coil current in the feedback loop

4.2 Servo control using current feedback

The electro-dynamical transducer generates a back EMF defined by $Bl(x)v$ at electrical side, depending on the instantaneous force factor $Bl(x)$ and the voice coil velocity $v=dx/dt$. Connecting the transducer to a low impedance source (voltage drive) the back EMF effects the measured voice coil current. Thus an electro-dynamical transducer may be used as actuator and sensor at the same time. However, the relationship between velocity v and current i is nonlinear due to the force factor variation $Bl(x)$ versus x . A feedback of the distorted current induces additional distortion into the loudspeaker. The same problem occurs if the back EMF is separated by a bridge arrangement or monitored by an additional sensing coil [4] as long as a constant force factor $Bl(x)=Bl(0)$ can not be guaranteed.

Thus, using a real loudspeaker as sensor itself requires a detector performing a nonlinear processing of the back EMF [29] as shown in Fig. 3. However, the design of the nonlinear detector requires a reliable physical model and means for measuring the large signal parameters precisely. This technique can not be realized by using analogue electronics only and will be discussed in greater detail below.

4.3 Current drive

Instead of driving the loudspeaker with a defined voltage provided by a low impedance source it is also possible to drive the loudspeaker

with a current source controlled by the input signal [8]. The current source has a much higher output impedance than the input impedance of the loudspeaker. Thus, variations of the voice coil inductance $L_e(x)$ versus displacement x have no effect on the input current i and the mechanical driving force $Bl(x)i$. However, the practical benefit of current drive is questionable. The reluctance force

$$F_{rel} = -\frac{1}{2} \frac{dL_e(x)}{dx} i^2. \quad (4)$$

which is a second effect of the nonlinear inductance $L_e(x)$, and other mechanical or acoustical nonlinearities can not be compensated by current drive. The variation of $L_e(x)$ versus x may be reduced by using a shorting ring at the optimal place. This is a cost effective solution which solves the cause of the problem.

5 Linearization with generic adaptive filters

The analogue linearization techniques fail if there is time delay generated between loudspeaker input and sensor output. For example, in horn loaded compression drivers the sound pressure signal measured at a point where the wave steepening is completed can not be used as a feedback input without causing an instability in the loop. In this case a linearization can only be accomplished as a feed-forward controller as shown in Fig. 4. No state variable measured at the loudspeaker is used as a feedback signal. The transfer behavior of the controller can be adjusted by the parameter vector \mathbf{P} provided by an adaptive detector circuit. The detector circuit estimates the optimal controller parameters by using the input signal $u(t)$ and the output signal $p(t)$. This arrangement guarantees self-tuning and an optimal adjustment of the controller while reproducing an audio signal. However, the detector may be deactivated at any time and the controller stays operative like a filter using the stored parameters \mathbf{P} .

Both the controller and the detector use a nonlinear feed-forward model of the loudspeaker. If no information about the physics of the nonlinear mechanisms is available then a generic model has to be used. Nonlinear control and system identification provide a large variety of possible candidates [10] - [20]. In the application to loudspeakers the following requirements have to be considered:

- Stability of the feed-forward controller and the adaptive parameter update system,
- handling of substantial time delay in the transducer/sensor system,
- the memory of the generic model should cover the impulse response of the loudspeaker,
- modeling of higher-order distortion,
- optimal parameter adjustment preventing stalling at local minima.

A detector meeting these requirements is depicted in Fig. 4. It comprises a linear adaptive FIR filter with the transfer function $H_{lin}(s)$ connected in parallel to a nonlinear adaptive filter with a linear weight network. The output of the linear filter

$$y_l = \mathbf{P}_l^T \mathbf{x} \quad (5)$$

is the scalar product of the linear parameter vector \mathbf{P}_l and the state vector

$$\mathbf{x}^T = [u(t), u(t-\tau), u(t-2\tau), \dots, u(t-t_i)]. \quad (6)$$

comprising the time-delayed samples of the time-discrete voltage signal.

The nonlinear filter expands the state vector \mathbf{x} into a nonlinear state vector \mathbf{A} by using, for example, a polynomial expansion such as

$$\mathbf{A}^T = T(\mathbf{x}) = [x_1 x_1, x_1 x_2, \dots, x_1 x_N, x_2 x_3, \dots, x_3 x_4, \dots, x_N x_N] \quad (7)$$

or a neural network with fixed parameters in the hidden layers. Linear weighting of \mathbf{A} with the nonlinear parameter \mathbf{P}_n gives the output of the nonlinear filter

$$y_n = \mathbf{P}_n^T \mathbf{A}. \quad (8)$$

To estimate the parameters in both filters the error signal

$$e = p - p_l - p_n \quad (9)$$

is calculated as the difference between loudspeaker output $p(t)$ and the sum of both model outputs. Searching for the minimum of mean squared error

$$MSE \equiv J = E[(e(t))^2]. \quad (10)$$

where the gradient of J versus the parameters becomes zero result in the steepest-descent algorithm providing optimal estimates on the linear parameter vector

$$\mathbf{P}_l(i+1) = \mathbf{P}_l(i) + \mu e \mathbf{x} \quad (11)$$

and the nonlinear parameter vector

$$\mathbf{P}_n(i+1) = \mathbf{P}_n(i) + \mu e \mathbf{A}. \quad (12)$$

The nonlinear controller in Fig. 4 contains a copy of the nonlinear filter provided with the nonlinear parameters \mathbf{P}_n . The following linear filter with the inverse transfer function $H_{lin}(s)^{-1}$ compensates for the linear transfer of the synthesized distortion $d(t)$ through the loudspeaker to accomplish a cancellation of the real distortion at the loudspeaker output.

Using adaptive filters with a feed-forward structure have some disadvantages:

1. If the amplitude of the synthesized distortion $|d(t)|$ are not small then the input signals $v(t)$ and $u(t)$ at the two nonlinear feed-forward system will differ and will produce a mismatch between the synthesized and real distortion. This limits the application of indirect parameter updating to medium amplitudes.
2. The compensation of third- and higher-order distortion requires a large number of nonlinear parameters. Since the model has to be implemented in the controller as well as in the detector this approach will be limited to second- and third-order polynomial expanders to make the implementation feasible.
3. The linear modeling of a loudspeaker reproducing the full audio band requires a linear FIR with many parameters. The

- number of the nonlinear parameters in a polynomial filter of n th-order will rise with the n th-power of the taps of the linear filter.
- 4. The parameters and state variables used in a generic structure does not directly correspond with the electrical, mechanical or acoustical mechanisms in a loudspeaker and can not be used to protect a loudspeaker under overload conditions.
- 5. The input-output linearization based on generic structures requires that the output signal can be observed and measured by a sensor. Observing an internal state variable (e.g. input current) and linearizing the output signal (e.g. sound pressure) at the same time is not possible because the relationship between the internal state and the output is unknown.

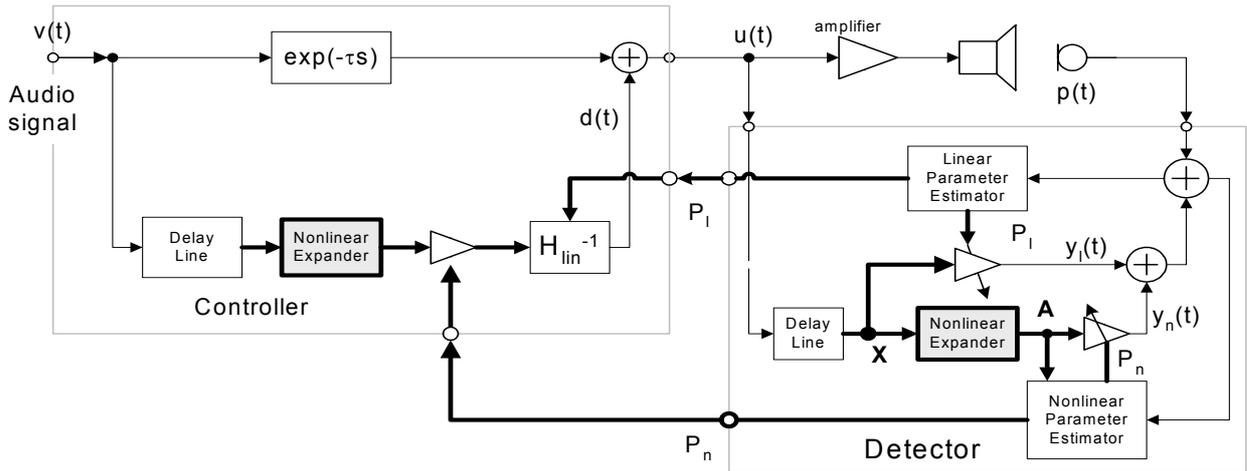


Fig. 4: Adaptive indirect control of a loudspeakers using a generic feedforward structure both in controller and detector

6 Linearization based on loudspeaker modeling

The results of loudspeaker modeling in the large signal domain revealed detailed information about the distortion generation which can be used to compensate loudspeaker distortion in a most effective way. However, this approach leads to special controllers depending on the transducer principle, the radiation aids used and intended application.

6.1 Autonomous subsystems connected in series

The discussion of the general signal flow chart in Fig. 1 revealed that the relationships between single input and multiple output can only be linearized if all nonlinearities may be concentrated in a nonlinear subsystems h_i with $i=1, \dots, N$ connected in a series as shown in Fig. 5. The relationship between the signal $y_{N+1}(t)$ and the multiple outputs $p(r_i)$ in the sound field should be linear transfer function $H_i(s)$. The AC-coupled amplifier and the passive crossover are considered in the linear system $H_0(s)$ between audio signal $u_o(t)$ and the signal $y_1(t)$ at the terminals of the electro-mechanical transducer.

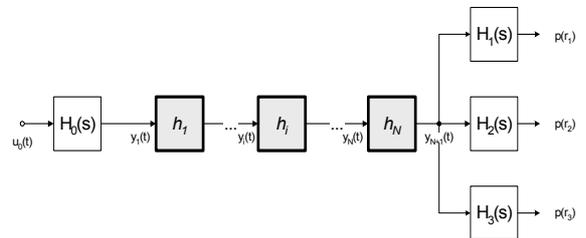


Fig. 5: Basic modeling of a loudspeaker by using linear and nonlinear subsystems

If the nonlinearities in the electrical, mechanical or acoustical domain are only related via the state variables y_i for $i=1, \dots, N$ they may be separated in autonomous subsystems. For example, the motor nonlinearities which are part of the subsystem h_1 have to be separated from the nonlinear radiation (Doppler effect) in a successive subsystems h_2 provided with voice coil displacement $y_2(t)=x(t)$ [24]. The nonlinear sound propagation may be modeled by the following subsystems h_3, \dots, h_N . The wave steepening in horn loaded compression drivers is modeled in [25] by such a series of nonlinear subsystems representing successive sections of the horn. The nonlinearity in each horn section has a distinct asymmetrical characteristic and produces second-order distortion predominantly but the following sections will transform the second-order distortion into third- and higher-order distortion.

Contrary, the dominant nonlinearities in electro-dynamical transducers such as force factor $Bl(x)$, voice inductance $L_e(x)$ and mechanical

compliance $C_{ms}(x)$ interfere with each other via the displacement x and can not be separated in two autonomous subsystems.

A controller connected to the input of the loudspeaker modeled in Fig. 5 gives a linear input-output relationship of the overall system if the controller has the following properties:

1. If the loudspeaker model comprises N nonlinear subsystem then there exist for each subsystem h_i with $i=1, \dots, N$ a corresponding nonlinear subsystem g_i $i=1, \dots, N$ and a linear subsystem G_{i-1} in the controller to ensure that the time delayed output signal

$$u_i(t - \tau_i) = y_i(t) \quad i=1, \dots, N \quad (13)$$

of each nonlinear control system g_i is equal with the corresponding input signals y_i of the nonlinear subsystem h_i of the loudspeaker model.

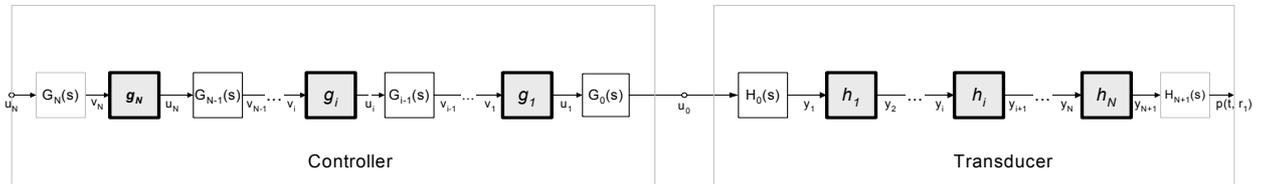


Fig. 6: Active linearization of a loudspeaker comprising separated nonlinearities

6.2 Linearization of an autonomous subsystem

The complex task of linearizing a loudspeaker with multiple nonlinearities requires the independent linearization of each subsystem h_i by a nonlinear control law g_i which transforms the input signal v_i into the output signal u_i by using an internal state vector \mathbf{x}'_i . The internal states \mathbf{x}'_i in nonlinear control subsystem g_i correspond with the internal states \mathbf{x}_i in the subsystem h_i of the loudspeaker model

$$\mathbf{x}'_i(t - \tau_i) = \mathbf{x}_i(t) \quad i=1, \dots, N \quad (15)$$

If there is any time delay ($\tau_i > 0$) between controller output u_i and transducer input y_{i+1} for $i=1, \dots, N$, then the controller states \mathbf{x}'_i can not be measured directly but have to be synthesized by using the controller input u_{i+1} or output u_i .

The structure of the control law g_i and the vector \mathbf{x}'_i depends on the physical mechanisms in subsystem h_i and the mathematical tools used in the modeling:

6.2.1 Integro-differential equation

The relationship between input y_i and output y_{i+1} of the system h_i can be described by an integro-differential equation using natural state variables of the physical system such as current, displacement, sound pressure in vector \mathbf{x}'_i . The nonlinear operations are performed in the time domain by performing multiplications between state variables (e.g. displacement x) and the nonlinear parameters (e.g. force factor $Bl(x)$) which are static nonlinear functions without any memory. Some of the linear dynamics may be separated from the nonlinear dynamics by using linear system functions which are transformed via the inverse Laplace transform into the time domain. This representation is very

2. The linear subsystem

$$G_0(s) = H_0(s)^{-1} \exp(-s(\tau_0)) \quad (14)$$

at the output of the controller is the inverse of the linear filter $H_0(s)$.

3. A constant time delay τ_{i-1} for $i=1, \dots, N$ may be added to the linear system functions $G_{i-1}(s)$ to make the system causal.

Any linear system $G_N(s)$ connected to the input of the controller and the linear system $H_{N+1}(s)$ after the nonlinear subsystems in the loudspeaker model have no influence on the linearization of the output signal.

useful for modeling acoustical processes with time delay and high number of modes (e.g. horn propagation and vibrations on a panel).

Using this representation the control law g_i can be derived by the following steps:

1. Representing the nonlinear subsystems between control output u_i and transducer output y_{i+1} by an integro-differential equation \mathbf{E}_T
2. The desired linear overall system between control input v_i and transducer output y_{i+1} is represented by an integro-differential equation \mathbf{E}_D .
3. Combining equations \mathbf{E}_T and \mathbf{E}_D gives the control law between control input v_i and control output u_i .
4. The state variables used in the control law are generated from the control input v_i or control output u_i depending on the state generation in h_i .

This technique has been used for the derivation of special mirror filters compensating for dominant driver nonlinearities, Doppler effect [24] and nonlinear sound propagation [25]. This approach is powerful and general but requires some intuition and experience in representing the equation \mathbf{E}_T and \mathbf{E}_D in an appropriate form. The resulting control law comprises a minimal number of terms which can be implemented in a digital processing system by using linear filters, static nonlinearities (look-up tables, power series expansions), adders and multipliers. Each term compensates for a separate nonlinear effect and is interpretable from a physical point of view.

6.2.2 State-space representation

Suykens [27] suggested to use the standard geometrical control technique to derive the inverse dynamics (ID-controller) and the linear dynamics (ID-controller) from the state-space model of h_i . Instead of using two separate controller parts to compensate for the complete dynamics first and then to reintroduce the desired linear dynamics it is preferable to derive the control law in the direct form [29]:

1. The nonlinear subsystems between control output u_i and transducer output y_{i+1} is represented by a state space model \mathbf{S}_T using the state vector \mathbf{x}'_i .
2. Transformation of the state space model \mathbf{S}_T into the normal form \mathbf{S}_N by using a diffeomorphism T which transforms \mathbf{x} into a new state vector \mathbf{z} .
3. The desired overall system between control input v_i and transducer output y_{i+1} is represented by a linear state space model \mathbf{S}_D in normal form using state vector \mathbf{z} .
4. Combining the state space models \mathbf{S}_N and \mathbf{S}_D gives the control law in the direct form between control input v_i and control output u_i .
5. The state variables \mathbf{x}'_i can be measured at the transducer directly if there is no time delay τ_i between the control output u_i and system input y_i (commonly known as static state feedback control). In most loudspeaker applications the state vector \mathbf{x}'_i has to be estimated by using an observer or a state predictor.

The control law is completely static in the state variables \mathbf{x}'_i and may be realized by using only static nonlinearities, adders and multipliers. Unlike the control law derived from the integro-differential equation there are no additional linear systems (differentiators, filters). However, despite differences in the form both control laws are identical in principle and allow perfect linearization of the modeled nonlinearities. The main advantage of this approach is that if the system h_i can be represented by a state space model, then formal tools can be used to derive the control law and to check its stability. Unfortunately, this approach is less convenient if there is unmodelled dynamics (e.g. vibration on a panel) involved, the number of modes and state variables is high and time delay has to be considered. For this reason this approach has been applied to woofers only.

6.2.3 Volterra Model

Kaizer [22] suggested to model the loudspeaker under voltage and current drive by a Volterra series and derived analytical expression for

the linear, second-order and third-order system function from the differential equation. Using this approach a control law may be derived in the following way:

1. The nonlinear subsystems between control output u_i and transducer output y_{i+1} is modeled by a Volterra series expansion of nth-order based on an analytical representation of the linear and higher-order system functions $H_i(s_1, \dots, s_i)$ with $i=1, \dots, n$.
2. The control g_i is modeled by a Volterra series expansion of nth-order comprising a linear, quadratic and higher-order homogenous power system connected in parallel. Each homogenous power system is represented by the system functions $G_i(s_1, \dots, s_i)$ with $i=1, \dots, n$.
3. The transfer function $H_i(s_1, \dots, s_i)$ is transformed into the inverse system functions $G_i(s_1, \dots, s_i)$ for $i=1, \dots, n$ by using

$$G_i(s_1, \dots, s_i) = \frac{-H_i(s_1, \dots, s_i)}{H_1(s_1)} \quad (16)$$

4. Based on the analytical structure of $G_i(s_1, \dots, s_i)$ a discrete nonlinear system is synthesized comprising linear systems (filter, differentiator) static nonlinearities, adders and multipliers.

This approach results in a consequent feed-forward structure where the states are synthesized implicitly from the control input v_i . This guaranties stability of the control system g_i for any choice of the parameters. However, the higher-order systems functions comprise a large number of terms which can not be implemented in available digital platforms. Controllers using only second- and third-order systems functions only provide an "approximative" linearization of h_i limited to low and medium amplitudes.

6.3 Example: Electro-mechanical transducer

The derivation of the control law is discussed on the electro-mechanical transducer using the electro-dynamical principle. This example is chosen because the mirror filter design and classic control design lead basically to the same control law giving exact linearization of the modeled nonlinearities. The main difference in the two approaches is the generation of the state variables \mathbf{X} , the stability and robustness of the controller and the suitability for adaptive parameter adjustment resulting to a self-learning system.

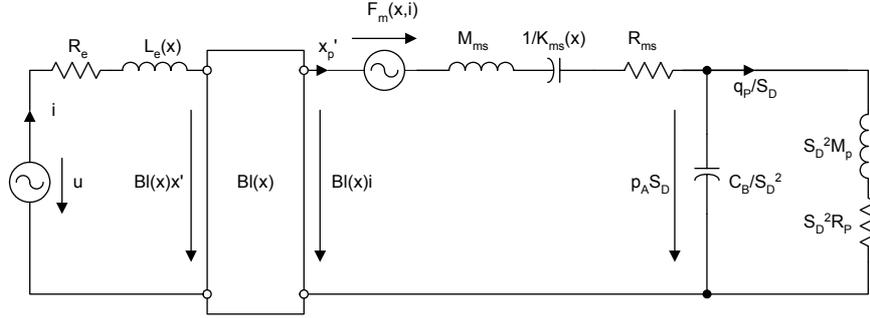


Fig. 7: Equivalent Circuit of the vented-box loudspeaker system

6.3.1 Equivalent Circuit

At low frequencies the electro-mechanical transducer may be modeled by an equivalent circuit with lumped elements because the wavelength is large in comparison to the geometrical dimensions. The equivalent circuit of a vented-box system is shown in Fig. 7. The DC-resistance R_e and the voice coil inductance $L_e(x)$ and the back EMF represent the electrical domain while neglecting irregularities of the input impedance (para-inductance) caused by the generation of eddy currents in the pole pieces and the voice coil former. For an electro-dynamical motor without shorting ring or copper cap on the pole piece the nonlinear parameter $L_e(x)$ has a distinct asymmetric characteristic giving higher inductance for negative voice coil displacement (moving towards the back-plate). The force factor $BI(x)$ represents the electro-dynamical coupling between electrical and mechanical side. The $BI(x)$ has a symmetrical characteristic if the magnetic induction B is uniform in the gap, there is a symmetrical fringe field and the coil is at optimal rest position. The variation of $BI(i)$ versus voice coil current i causes an effect called flux modulation. However, the variation of $BI(i)$ is negligible as long as the static part of the induction B generated by the magnet is much larger than the AC part generated by the voice coil current i . On the mechanical side the electro-dynamical driving force $BI(x)i$ and electro-magnetic driving force giving in Eq. (4) excite the mass-spring system and the acoustical resonator transformed to the mechanical domain. The stiffness of the mechanical suspension is assumed as nonlinear depending on the voice coil displacement x . Nonlinearities of the R_{ms} are neglected because the electrical damping is usually dominant over the mechanical damping in loudspeakers connected to a voltage source.

6.3.2 State space representation

The nonlinear differential equation of the analogous circuit written in the general state space form is

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{a}(\mathbf{x}) + \mathbf{b}(\mathbf{x})y_1 \\ y_2 &= \mathbf{h}(\mathbf{x}) \end{aligned} \quad (17)$$

where $y_1 = u$ is the voltage at the terminals of the voltage driven loudspeaker, y_2 is the output corresponding to the voice-coil displacement x . The state vector of the system $\mathbf{x}(t) = [x_1, x_2, x_3, x_4, x_5]^T = [x, dx/dt, i, q_p, p_A]^T$ comprises displacement x and velocity $v = dx/dt$ of the voice-coil, the electrical input current i , the volume velocity q_p in the port and the sound pressure p_A in the acoustic system. The components $\mathbf{a}(\mathbf{x})$, $\mathbf{b}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ are smooth nonlinear functions of the state vector \mathbf{x} specified for the vented-box system in [29, Eq. 3-5]. The state space model in Eq. (17) is illustrated by a signal flow chart depicted in Fig. 8. The nonlinear functions $\mathbf{b}(\mathbf{x})$ and $\mathbf{a}(\mathbf{x})$ and the

integrator are part of a feedback loop generating the state vector \mathbf{x} . The distortion react to its own generation process forming a feedback loop. If the nonlinearities in $\mathbf{a}(\mathbf{x})$ is only a simple squarer generating second-order harmonic and intermodulation distortion primarily the feedback of the distortion to the same squarer will also produces third- and eventually all kinds of higher-order distortion. The feedback also explains instabilities (coil jump out effect), the complicated relationship between input/output amplitude (nonlinear amplitude compression) and the interactions between different driver nonlinearities. Any generic filter with a feed-forward structure (polynomial filter) can only approximate the behavior of the electro-mechanical transducer in a limited amplitude range.

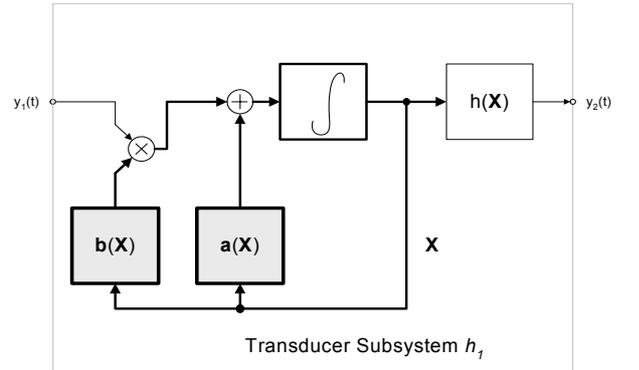


Fig. 8: State space model for the electro-mechanical transducer

6.3.3 State space representation in normal form

To linearize the subsystem h_1 a direct relationship between input $y_1(t)$ and output $y_2(t)$ is required. Following the general approach of geometrical control theory [31] the output $y_2(t)$ is differentiated until the system input $y_1(t)$ appears

$$y_2^{(\gamma)} = L_a^\gamma h(\mathbf{x}) + L_b L_a^{\gamma-1} h(\mathbf{x}) y_1 = f(\mathbf{x}) + g(\mathbf{x}) y_1 \quad (18)$$

where $L_a h$ and $L_b h$ stand for the Lie derivatives of $h(\mathbf{x})$ along the smooth vector fields $\mathbf{a}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$, respectively. The number γ of differentiation required to have $L_b L_a^i h(\mathbf{x}) \neq 0$, for $i=0, \dots, \gamma-2$, and

$L_b L^{\gamma-1} a h(\mathbf{x}) \neq 0$ for all \mathbf{x} is called the relative degree of the system. It corresponds with the excess of poles over zeros in a linear system.

The output signal y_2 and its derivatives $y_2^{(i)}$ for $i=1, \dots, \gamma-1$ may be considered as new state variable z_1, \dots, z_γ . If the relative degree γ is smaller than the order n of the system than the system has additional internal dynamics represented by additional state variables $z_{\gamma+1}, \dots, z_n$ which are not directly controllable from the input y_1 and observable at the output y_2 . The old and the new state vector are related by a smooth, locally defined coordinate transformation (diffeomorphism) which has also a smooth inverse.

Using the diffeomorphism $T: \mathbf{x} \rightarrow \mathbf{z}$ Eq. (18) is expressed in the state space representation in normal form

$$\begin{aligned}
 \dot{z}_1 &= z_2 \\
 \dot{z}_2 &= z_3 \\
 &\dots \\
 \dot{z}_\gamma &= f(T^{-1}(\mathbf{z})) + g(T^{-1}(\mathbf{z}))y_1 = f(\mathbf{x}) + g(\mathbf{x})y_1 \\
 \hline
 \dot{z}_{\gamma+1} &= I_1(\mathbf{z}) \\
 &\dots \\
 \dot{z}_n &= I_{n-\gamma}(\mathbf{z}) \\
 \hline
 y_2 &= z_1
 \end{aligned} \tag{19}$$

using the function $I_1(z), \dots, I_{n-\gamma}(z)$ which describe the internal dynamics.

The vented-box system has the order $n=5$ and a strong relative degree $\gamma=3$ where the first three elements of the state vector \mathbf{z} are the displacement $z_1=x$, velocity $z_2=v$ and acceleration $z_3=a$. If the input y_1 is used to control the displacement and its derivative to zero $z_1=z_2=z_3=0$ then the two internal states variables $z_4=q_p$ and $z_5=p_A$, volume velocity in the port and sound pressure, respectively, describe the zero dynamics in the acoustical (Helmholz) resonator. Due to the acoustic resistance R_p any vibration in the zero dynamics vanishes exponentially. This is an important criteria for the stability of the system.

The state space representation of the closed box system in normal form is illustrated by the signal flow chart in Fig. 9. The nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$ given in [29, Eq. 9,10] produce scalar outputs which are multiplied with and added to the input signal giving the acceleration of the driver. A series of first-order integrators generate the state variables z_2 and the output signal $y_2=z_1=x$. The feedback loop is closed by the diffeomorphism $T^T: \mathbf{z} \rightarrow \mathbf{x}$ transforming the state variables back into the old parameters. The zero dynamics is also a feedback loop separated from the input/output path and generating the additional states z_4 and z_5 .

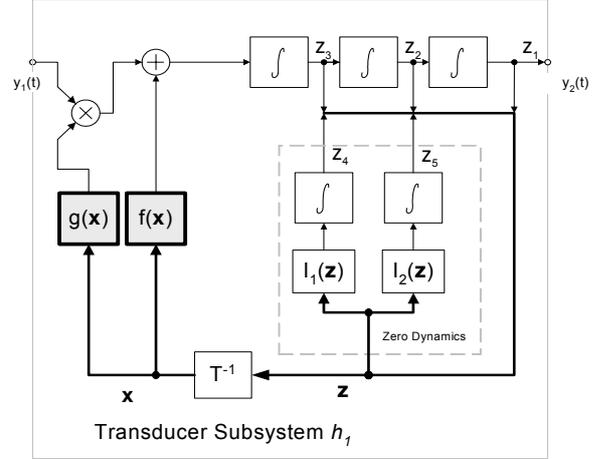


Fig. 9: State space model of a loudspeaker with radiation aids (e.g. vented enclosure) in normal form.

6.3.4 Linearization with separate ID and LD-Controller

Having the transducer system transformed into the normal state space form the system can be linearized by compensating the effect of the nonlinear system $g(\mathbf{x})$ and $f(\mathbf{x})$ by a controller with inverse dynamic (ID)

$$u_1 = \frac{w - f(\mathbf{x})}{g(\mathbf{x})} \tag{20}$$

giving the overall relationship $y_1^{(i)}=w_i(t)$. Thus, applying the ID-controller to a closed-box system the displacement rises to lower frequencies by 18 dB per octave. An additional controller with linear dynamics (LD)

$$w_1 = v_1 g_D + f_D(\mathbf{z}) \tag{21}$$

is connected in front of the ID-controller to limit the displacement below the Helmholtz resonance, to preserve a desired system alignment and to have a constant amplitude sound pressure response above the resonance frequency. Applied to a vented-box system the control law with the function $f_D(\mathbf{z})$ and $g(\mathbf{x})$ given in [29, Eq. 16, 17] produces a linear overall system equal with the system function $H_s(s)$ using the linear loudspeaker parameters [29, Eq. 18,19].

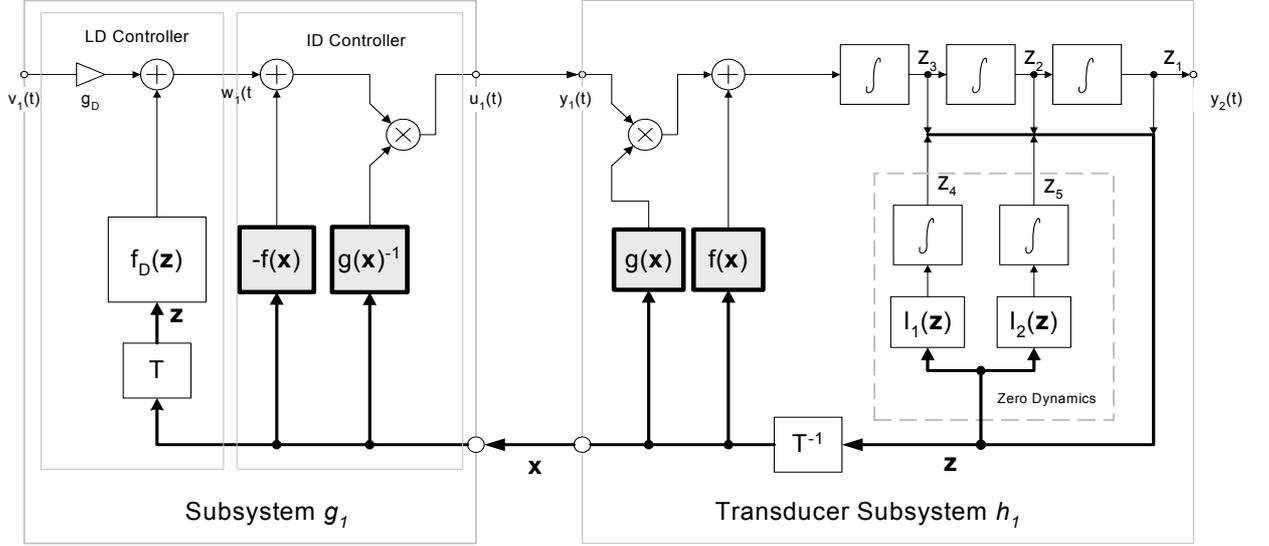


Fig. 10: Perfect linearization by static state feedback with a separate controllers for the linear and inverse nonlinear dynamics

6.3.5 Control Law in Direct Form

The linearization of a transducer using separate LD and ID controllers requires full access to all state variables in vector \mathbf{x} and sufficient processing capacity to transform $\mathbf{x} \rightarrow \mathbf{z}$ and to calculate the $f(\mathbf{x})$, $g(\mathbf{x})$, $f_D(\mathbf{z})$. Although, the compensation of the entire linear and nonlinear dynamics and the replacement of the desired poles is the standard approach in nonlinear control it is advantageous to summarize the LD and ID controller in Eqs. (20) and (21) to the control law in the direct form [29]

$$u_1 = \frac{v_1 g_D + f_D(\mathbf{z}) - f(\mathbf{x})}{g(\mathbf{x})} = \frac{v_1 - \beta(\mathbf{x})}{\alpha(\mathbf{x})}. \quad (22)$$

The control law for the vented-box system is illustrated in Fig. 11. The transducer contains a linear model in the integrator-decoupled form where the terms $f_D(\mathbf{z})$ and g_D and the zero dynamics $I_1(\mathbf{z})$ and $I_2(\mathbf{z})$ determine the desired transfer function $H_x(s)$. The nonlinear terms $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$ in the feedback loop generate the undesired distortion when the loudspeaker is operated in the large signal domain. The controller comprises only the inverse terms $-\beta(\mathbf{x})$ and $\alpha(\mathbf{x})^{-1}$ which supply "synthesized" distortion to the control input to compensate the real distortion in the transducer model.

The direct form of the control law in Eq. (22) has the following advantages :

- If the loudspeaker is operated in the small signal domain and the amplitudes of the state variables in \mathbf{x} are small then

the control input becomes identical with the output because $\beta(\mathbf{x}) = 0$ and $\alpha(\mathbf{x})^{-1} = 1$. This is a nice feature for the implementation on inexpensive signal processors using a fixed-point arithmetic with limited resolution. Contrary, the output w_1 of the LD-controller in Fig. 10 requires 10 bit more resolution for the numerical representation of a digital signal covering the whole audio band.

- Whereas the ID controller requires full state measurement the direct control law for the electro-dynamical transducer requires only the displacement x , its velocity $v=dx/dt$ and the input current i as state information.
- The two internal states variables q_p and p_A , volume velocity in the port and sound pressure, respectively, and the linear parameters C_B , M_p and R_p of the acoustical resonator which are used in Eqs. (20) and (21) do not appear in Eq. (22).
- The control law is valid for the electro-dynamical transducer with dominant $Bl(x)$ -, $L_e(x)$ - und $C_{ms}(x)$ -nonlinearity mounted in a vented- or closed-box system or coupled with any radiation aid (horn, transmission line, passive radiator, distributed mode panel). All these acoustical elements contribute to the zero dynamics which does not appear in the control law in the direct form. Thus, the linearization of the output y_2 can also be accomplished also in cases where the zero dynamics is hard to model (e.g. high number of modes on a panel) or even nonlinear (e.g. nonlinear stiffness of a passive radiator).

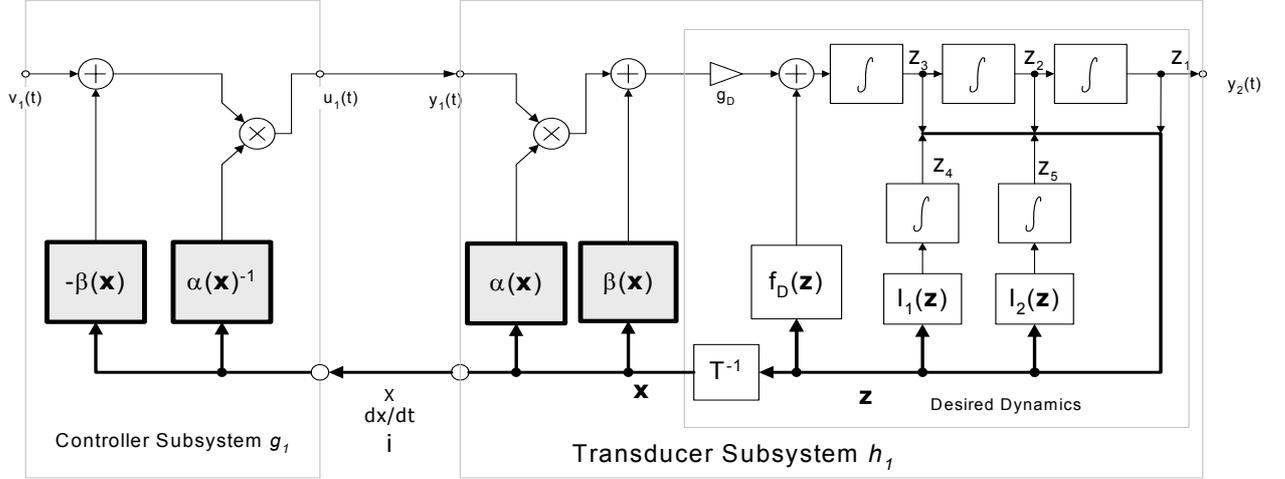


Fig. 11: Perfect linearization by static state feedback using a control law in the direct form

7 Providing the state information

7.1 Direct measurement

The direct control law in Eq. (22) requires the instantaneous state variables displacement x , velocity v and input current i . They may be measured directly on the electro-dynamical transducer by using displacement sensor and a current sensor and feed back to the control law as shown in Fig. 11. However, this approach has two major drawbacks:

1. Whereas the measurement of the current can be accomplished by a shunt there is no simple solution for the measurements of the mechanical signals. The voltage v may be generated by differentiating the voice coil displacement x . Unfortunately, the displacement can not be derived from the velocity or acceleration because the integration can not provide the DC-component of the voice coil displacement which is dynamically generated if the transducer nonlinearities are asymmetrical. A laser displacement sensor using the triangulation principle may be used for measuring displacement x under laboratory conditions but this solution is too expensive and impractical for common use.
2. Static state feedback based on direct measurement and feedback of the states variables does not allow any time delay in the sensor path. Since the realization of the nonlinear control law requires digital signal processing, special converters (DAC and ADC) are required. For example, the time delay produced in sigma-delta converters commonly used in audio applications may produce a phase shift between synthesized and real distortion which impairs the compensation and may also cause instabilities in the overall system.

7.2 State Observer

Beerling et. al. [26] suggested a nonlinear model to estimate the state variables of the transducer. Since the voice coil current can easily be measured by a shunt the other state variables may be estimated by an observer based on the voltage equation or nonlinear force equation [35]. The force equation requires knowledge of the mechanical

parameters and full modeling of the zero dynamics (acoustical resonator). A differentiator as shown in Fig. 12 generates the velocity from the displacement. The voltage equation only requires electrical parameters of the transducer (R_e , L_e , $Bl(x)$) to predict the velocity v and then generates the displacement x by integrating v . This method has the disadvantage that the DC part of the displacement can not be generated accurately.

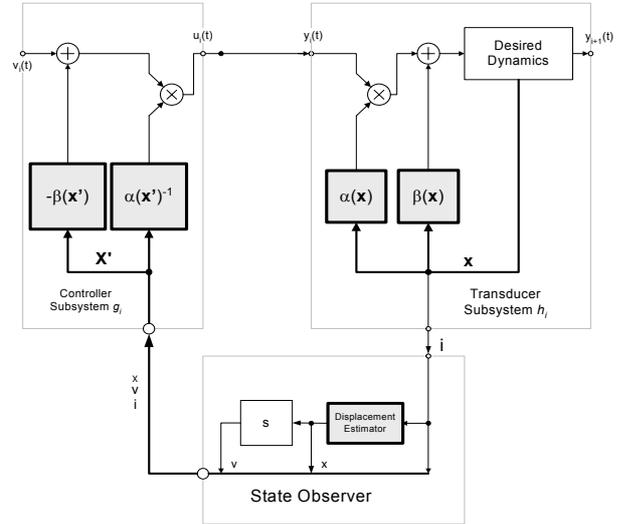


Fig. 12: Controller based on static state feedback controller with state observer using the measured voice coil current

However, it is more advantageous to dispense with any direct state measurement at the transducer and to synthesize the required state variables from the control output u_1 by using the state space model in Eq. (17) as shown in Fig. 13. Any time delay τ_i may be added between the control output u_1 and the transducer input y_1 .

The observer requires full knowledge of the linear and nonlinear transducer parameters and the zero dynamics. Using this control scheme with adaptive parameter estimation the internal feedback in the

observer and the external feedback via the control law may cause instabilities for some parameters settings. Even if the observer and the control law are stable as separate systems the stability of the feedback connection of both system may become unstable. Fortunately, the parameters both in the control g_i and in the observer have a common physical basis and may be checked for plausibility ($Bl(x) > 0$, $C_{ms}(x) > 0$, $M_{ms} > 0$).

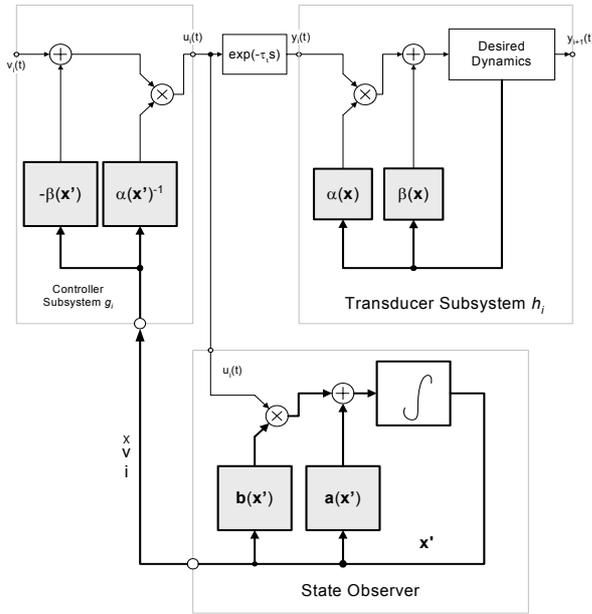


Fig. 13: State feedback controller using an observer for generating all state information from control output

7.3 State Prediction

The problems related with state observers may be avoided by generating the required state variables from the control input v_i as shown in Fig. 14. Applying the control law g_i to the transducer subsystem h_i the overall system between the control input v_i and the displacement $y_{i+1}=x$ becomes linear and can be described by the system function $H_x(s)$. Thus a linear filter with the system function $H_x(s)$ and a simple differentiator may be used to predict the displacement x and velocity v directly. The linear filter also comprises the zero dynamics (caused by the enclosure or other radiation aids). Only the generation of the input current i requires a nonlinear current estimator [29, Eq. 30] and a linear filter [29, Eq. 26]. The state predictor and the control law result in a feed-forward structure which is just the mirror image of the transducer model comprising the inverse elements in a feedback loop. The feed-forward structure guarantees stability of the overall system as long as the linear filters and the nonlinear current estimator are stable. This control scheme is an ideal candidate for adaptive control.

Active Compensation of Transducer Nonlinearities

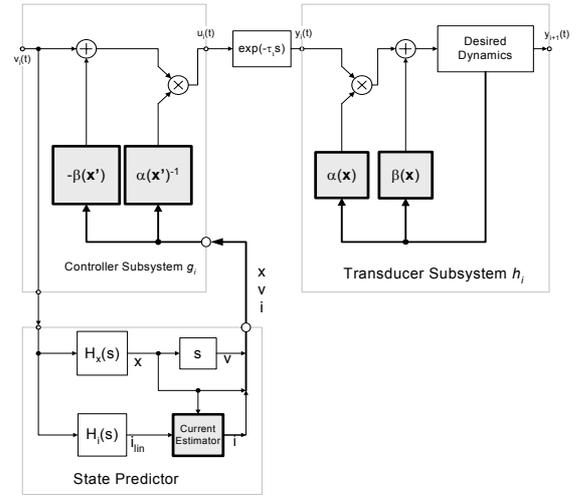


Fig. 14: Linearization of the electro-mechanical transducer using a feedforward controller based on state prediction (mirror filter)

8 Adaptive Control

The success of the linearization based on transducer modeling depends on two conditions:

1. Adequate modeling of the nonlinear mechanisms in the subsystem h_i .
2. Optimal adjustment of the free parameters of the controller to the particular transducer subsystem h_i .

Whereas the first condition has to be proven once for the loudspeaker type used, the second condition has to be ensured for any unit for the full time of usage. Adaptive updating of the free control parameters is an interesting way for realizing a self-learning device and for compensating for the influence of climate variation (temperature and humidity) and aging of the material. However, adaptive algorithm raises the following requirements:

- Updating should dispense with an artificial test stimulus but should stay operative for most audio signals.
- A state variable of the transducer has to be monitored which is easy accessible.
- The sensor should be inexpensive, robust and precise.
- The update algorithm itself should be stable for any choice of the parameters.
- The update algorithm should be robust in cases of parameter uncertainties and un-modeled dynamics.

The adaptive control techniques may be separated into the direct and indirect methods:

8.1 Direct updating

Adjusting the parameters in the controller by searching for minimal distortion in the transducer output y_{i+1} and linearizing the overall system is called direct updating. An error signal $e(t)$ is generated by

comparing the linear output $y_{i+1}'(t)$ with the measured output $y_{i+1}(t)$ that reflects the instantaneous nonlinear distortion [36]. A minimum of the mean squared error ensures an optimal adjustment of the controller.

Fig. 15 shows the direct adaptive adjustment of the parameters of the control law with state prediction (mirror filter). The linear parameters of the state predictor are separately adjusted by using a linear adaptive filter $H_x(s)$ connected in parallel to the overall system and straightforward adaptive techniques [21]. The nonlinear parameters may be directly adjusted in the control law. The control law g_i is parameterized by using a nonlinear expander generating the nonlinear states \mathbf{A} scaled by the weights \mathbf{P} giving

$$u_1 = v_1 + \mathbf{P}^T \mathbf{A}. \quad (23)$$

All of the unknown parameters are collected in parameter \mathbf{P} while the elements of \mathbf{A} are known nonlinear functions of x , v and i .

The parameters \mathbf{P} may be updated by using an intermittent filtered-gradient LMS algorithm [34, Eqs. 34, 35]

$$\mathbf{P}_n(i+1) = \mathbf{P}_n(i) + \mu e(t) \left(h_x(t - \tau_i) * \mathbf{A}(t) \right)_{|e| < e_{max}} \quad (24)$$

where $h_x = L^{-1}\{H_x(s)\}$ using the inverse Laplace transformation L and the convolution $*$.

The update process is interrupted when the error exceeds an allowed threshold (for example $e_{max} = 0.1 |y_{i+1}|$) and the linear gradient is likely to fail. At the beginning of the nonlinear update process when $\beta(\mathbf{x}) = 0$ and $\alpha(\mathbf{x})^l = 1$ then the nonlinear parameters are updated only at small and medium amplitudes of \mathbf{x} when the error $|e| < |e_{max}|$. With the progress of the updating process when the parameters \mathbf{P} approach to the optimal values \mathbf{P}^* the control law linearizes the overall system and the amplitude of the error decreases for all amplitude of \mathbf{x} . Finally, the condition $|e| < |e_{max}|$ holds for all \mathbf{x} and the parameters \mathbf{P} are updated permanently.

The intermittent filtered-error algorithm [34, Eqs. 36, 37] is an alternative techniques for updating the parameters directly. It requires only one filter for the inverse filtering of the error signal $e(t)$ and some time delay for each gradient signal in \mathbf{A} . However, this approach does not minimize the error in y_{i+1} but virtually in the control input v_i . This corresponds with a spectral coloration of the residual error signal e .

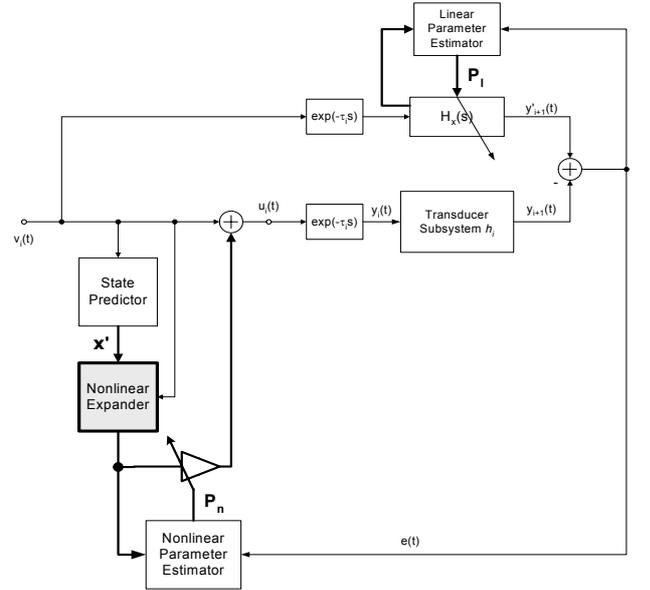


Fig. 15: Adaptive linearization of the electro-acoustical transducer using a nonlinear feedforward controller and a linear adaptive model $H_x(s)$

8.2 Indirect updating

More traditional approaches use indirect techniques where the loudspeaker is adaptively modeled and the estimated parameters are transformed into control parameters and copied into the control law. Clearly this approach requires more elements and processing power than direct updating of the control law. There are three ways to realize indirect updating:

8.2.1 Parallel Model

Here the adaptive model is connected in parallel to the subsystem h_i . The electro-mechanical transducer and other loudspeaker subsystems require a model having a feedback structure as shown in Fig. 8 corresponding with the state space model in Eq. (17). Since the model parameters are not linear in the output, the update algorithm [34, Eq. 19] might be trapped in local minima of the cost function.

8.2.2 Inverse Model

Alternatively, the adaptive model may be connected in series to the output of the transducer. It is the target to linearize the overall system between transducer input y_i and model output. The control law may be directly used as the inverse model system. It is an advantage of the inverse model that the free parameters are linear in the output and can be updated with straightforward methods [34, Eqs. 13, 14]. However, the adaptive updating of the inverse model will be biased if the loudspeaker output is corrupted by measurement noise.

8.2.3 Impedance model

A special form of indirect updating can be applied to electro-dynamical transducers. Here the nonlinear relationship between voltage $y_i = u(t)$ and current $i(t)$ at the transducer terminals as shown in Fig. 16. The impedance model is based on the voltage and force equation [35, Eqs. 13, 14] and generates an error signal $e(t)$ [35, Eq. 18] used for the

updating of the detector parameters \mathbf{P} . The parameters \mathbf{P} of the detector are closely related to the physics of the transducer and may be interpreted as linear parameters (resonance frequency f_s , loss factor Q_{ts}) and as relative nonlinear parameters $Bl(x/x_{max})/Bl(0)$, $C_{ms}(x/x_{max})/C_{ms}(0)$ and $L_e(x/x_{max})$ with the maximal displacement x_{max} . Clearly the mechanical parameters (M_{ms} , C_{ms}) and state variables (x , v) can not be identified as absolute quantities by performing only an impedance measurement and without having additional information about the mechanical system. This technique have been used for the dynamic measurement of the large signal parameters of electro-dynamical transducers [38].

The detector parameters \mathbf{P} satisfy the free parameters in the control law if expressed in the direct form and in the state expander as used in the mirror filter in Fig. 14. Since the parameter vector \mathbf{P} vary slowly due to varied ambient condition and aging the feedback of parameters is immune to any time delay τ_i in the signal path.

Monitoring the impedance at electro-dynamical transducers has the advantage:

- Inexpensive sensors for voltage and current (shunt or current sensors),
- no additional wiring,
- immune against ambient noise and acoustical environment,
- no time delay compensation required.

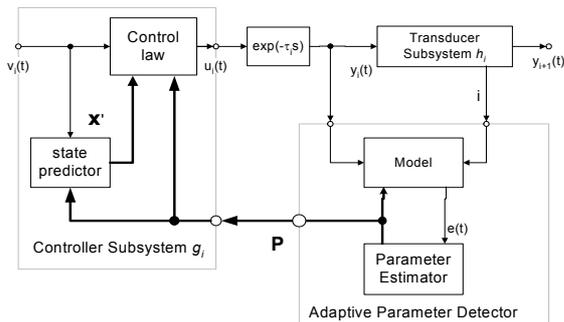


Fig. 16: Indirect adaptive linearization of an electro-mechanical transducer based on voltage and current measurement

8.3 Initial control information

The robustness of an adaptive control system can be increased by adjusting the degree of freedom of the control structure to the particular loudspeaker system. For instance a vented enclosure compared with the closed-box-system introduces additional zero dynamics which increases the order and the number of free parameters in the linear system $H_x(s)$ in the state predictor. The following information should be provided at the beginning of adaptive learning:

- type of the transducer principle (electro-dynamic or electro-static) and the application (woofer, mid-range or tweeter),
- the radiation aids used (sealed-box, vented-box, horn, panel),
- limit parameters describing the allowed mechanical or thermal overload.

9 Auxiliary Control Functionality

The control based on loudspeaker modeling provides information about the instantaneous state and parameters which may be used to expand the functionality and to enable an automatic control of a variety of different loudspeaker systems. The example in Fig. 17 shows such an automatic control system for a woofer system.

9.1 Diagnostics

Since the parameters in \mathbf{P} identified by adaptive control have a physical meaning they may be checked for plausibility. If the parameter updating fails for any reasons (wrong transducer type or loose electrical connection) the control may be deactivated and a user message can be generated. The interpretability of the control parameters may also be exploited to indicate a loudspeaker defect (voice coil offset). Storing the initial nonlinear compliance characteristic $C_{ms}(x)$ as a reference long term variation may be monitored and also a user message may be generated if aging or fatigue requires a replacement of the driver used in professional applications.

9.2 Memory

Since most of parameters \mathbf{P} measured by adaptive control varies slowly it is useful to store the parameters in a memory. Thus, restarting the controller the old parameters may be used as initial parameters in the adaptive control. The control law and the state predictor may also be operated as a non-adaptive controller with frozen parameters. This is important if limited processing power is available and multi-tasking is required. Implemented in a personal computer the adaptive updating in the loudspeaker controller may be activated when the processor is idling and the reproduced music signal gives sufficient excitation of the loudspeaker connected.

9.3 Protection system

Linearization of a loudspeaker makes only sense if there are also means provided to protect the speaker against thermal or mechanical damage because reducing the distortion in the loudspeaker output makes it also more difficult for the user to recognize a critical overload situation in time. The linear, nonlinear and thermal parameters identified by adaptive control make an effective protection of the loudspeaker possible:

9.3.1 Mechanical protection

In a passive loudspeaker without control the mechanical load of the suspension and the distortion generated in the output signal limit the maximal peak displacement X_{max} and the radiated sound pressure at low frequencies. The active linearization relaxes the limiting factor of the distortion. However, the maximal peak displacement X_{max} has to be limited to avoid a damage on the suspension system and to keep the power handling reasonable. The protection limits used in the large signal parameter measurement in the Distortion Analyzer [38] may also be used in a controller. If the minimal value of the ratio $C_{ms}(x)/C_{ms}(0)$ may be compared with an allowed limit value (e.g. $C_{lim}=20\%$) then the voice coil displacement has to be limited by an attenuator (e.g. adjustable high-pass). The linearization of the force factor distortion requires additional power for frequencies far away from the resonance frequency. Thus, it is reasonable to limit the displacement if the force factor ratio $Bl(x)/Bl(0)$ falls below a defined limit (e.g. $Bl_{lim}=25\%$).

9.3.2 Thermal protection

The increase of the voice coil temperature ΔT_v may be estimated from the changes of the input impedance. Whereas the adaptive impedance

detector may provide the instantaneous voice coil temperature the non-adaptive mode requires an additional thermal model using the thermal parameters of the loudspeaker. Due to the convection cooling and eddy currents generated in the pole tips there are many interactions between the mechanical and thermal mechanisms [40]. If the voice coil temperature ΔT_v exceeds a defined limit value the input signal have to be attenuated in some way.

9.4 Active compensation of voice coil offset

If the voice coil has an offset from optimal rest position the limited voice coil height increases the asymmetry of the $Bl(x)$ -curve which generates additional distortion and unstable behavior above the resonance frequency f_s . An offset is not only caused by a defect in driver manufacturing but can also be generated by fatigue or ageing of the suspension, wrong storage or the gravity acting on the voice coil if the driver is mounted in horizontal position (e.g. hat rest in a car).

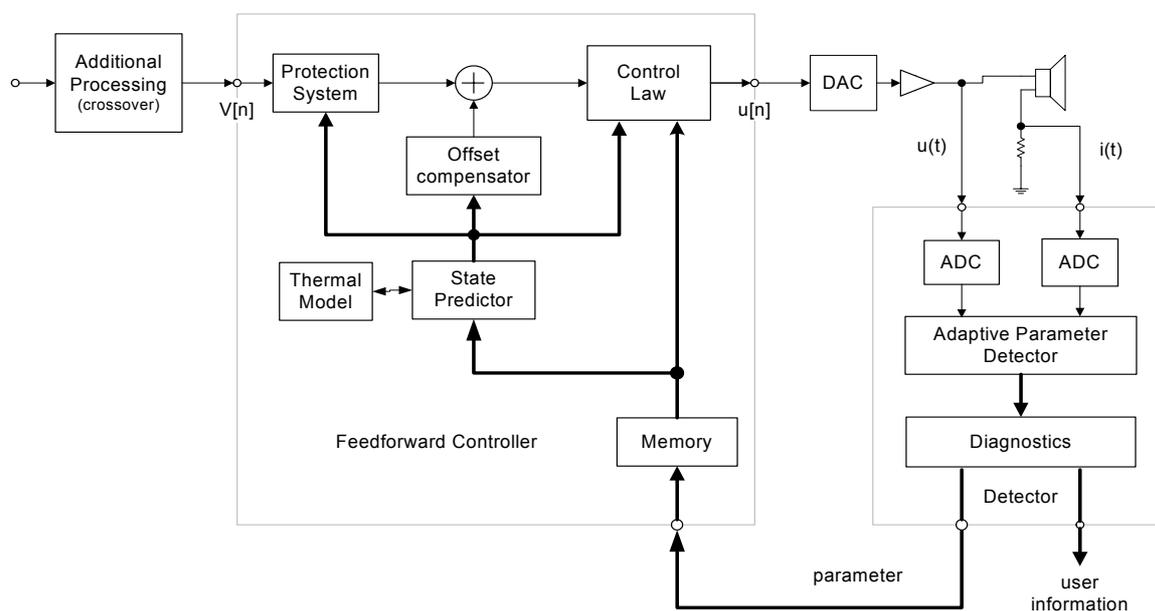


Fig. 17: Automatic control system for woofer

10 Conclusion

After discussing many technical details of active loudspeaker control the general goals, technical requirement and practical benefits shall be summarized:

Clearly active linearization and other forms of loudspeaker control competes with passive means available for loudspeaker improvements. The main goal of active loudspeaker control is not to compensate for loudspeaker defects (e.g. rub and buzz phenomena [42]) which can easily be fixed by improved loudspeaker design or manufacturing. Active loudspeaker control can only be justified in applications requiring smaller, less expensive transducers giving more output with lower distortion at higher sensitivity.

Since the dominant nonlinearities in loudspeakers can be modeled, control schemes which exploit this information are superior over generic controllers. Very important is the stability and robustness of the controller especially if the loudspeaker behaves irregular or there are any defects in the control system (loose connection). The controller should have self-tuning capabilities to simplify the adjustment to the

Although the control law compensates for the effect of the offset it is better to adjust the voice coil position actively. Knowing the offset in the $Bl(x)$ -characteristic a simple DC voltage is generated and supplied via the DC-coupled amplifier to the transducer. This becomes more and more interesting for loudspeakers with extremely small (sealed) enclosures where drivers with high compliance are preferred.

9.5 Additional Processing

Any additional signal processing commonly applied to audio signal such as active crossovers, equalizers and limiters may be applied to the input signal of the controller. Thus the controller, amplifier and transducer may be considered as unit having a defined (linear) transfer characteristic.

particular loudspeaker and to cope with heating or long term parameter variations. A sensor system used should be inexpensive and immune against ambient noise. Linearization and protection of a loudspeaker are closely related with each other. The protection system should prevent a damage for any input signal while causing minimal impact on the reproduced sound quality. The actively controlled loudspeaker should handle any signal without causing a thermal or mechanical damage. Since the nonlinear controller requires a defined transfer function between control output and loudspeaker input any form of signal attenuation or filtering (equalizer) should be performed prior to the nonlinear part.

Such a control system can not be realized in the analog domain. If a platform for digital signal processing is available loudspeaker control can be implemented as a software module requiring no or only minimal additional hardware (e.g. current sensor). Active loudspeaker control gives new freedom for the design of the passive transducers [32]. For example, the transducer will be optimized in respect with size, weight, sensitivity, bandwidth and directivity of the reproduced sound [33] but the linear transfer behavior and the regular loudspeaker distortion can also be controlled actively. Thus, there are new challenges in the

design of active loudspeaker systems which require a close cooperation between driver, system and controller development.

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