

Dynamical Measurement of the Effective Radiation Area S_D

Wolfgang Klippel¹ and Joachim Schlechter²

¹University of Technology Dresden, Germany, wklippel@klippel.de

²KLIPPEL GmbH, Dresden, Germany, schlechter@klippel.de

Abstract

The effective radiation area S_D is one of the most important loudspeaker parameters because it determines the acoustical output (SPL, sound power) and efficiency of the transducer. This parameter is usually derived from the geometrical size of the radiator considering the diameter of half the surround area. This conventional technique fails for microspeakers and headphone transducers where the surround geometry is more complicated and the excursion does not vary linearly versus radius. The paper discusses new methods for measuring the S_D more precisely. The first method uses a laser sensor and microphone to measure the voice coil displacement and the sound pressure generated by the transducer while mounted in a sealed enclosure. The second method uses only the mechanical vibration and geometry of the radiator measured by using a laser triangulation scanner. The paper checks the reliability and reproducibility of conventional and the new methods and discusses the propagation of the measurement error on the T/S parameters using test box perturbation technique and other derived parameters (sensitivity).

1. INTRODUCTION

Electro-acoustical transducers can be modeled at low frequencies by an equivalent circuit comprising a few lumped parameters. The effective radiation area S_D describes the coupling between the mechanical and the acoustical domain by replacing the cone, diaphragm, surround and other radiating parts by a rigid piston moving with the same velocity as the voice coil and having an equivalent surface generating the same sound pressure in the far field as the real radiator [1]. The parameter S_D determines the sound pressure on axis (e.g. pass band sensitivity) and the total acoustical output power [2]. A precise value of S_D is also a prerequisite for accurate parameter measurements using the test box perturbation technique and for providing reliable T/S parameters (e.g. equivalent air volume of driver stiffness V_{as}) for system design [3], [4].

In practice the effective radiation area is usually derived from the geometry of the radiating surface by reading the diameter d in the middle of the surround area as shown in Fig. 1.

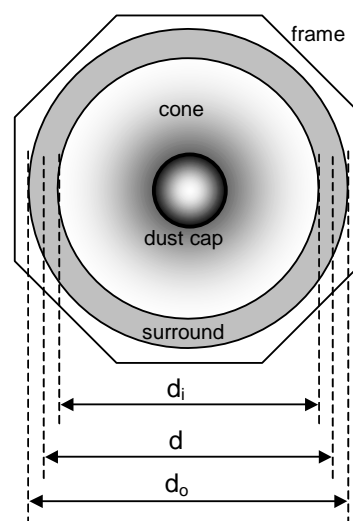


Fig. 1: Estimating the effective radiation area S_D from the radiator's geometry

This approach considers the cone moving as a rigid body and assumes a linear decay of the displacement

$$\frac{|x(\omega, r)|}{|x_{coil}(\omega)|} = \begin{cases} 1 & \text{for } 0 \leq r \leq d_i/2 \\ \frac{d_o - 2r}{d_o - d_i} & \text{for } d_i/2 \leq r \leq d_i/2 \end{cases} \quad (1)$$

versus radius r in the surround area. R. Small showed in the appendix of [5] that the diameter d referred to the half width of the surround is a useful approximation of

$$S_D = \frac{\pi}{4} \left(d_i^2 + (d_o - d_i)d_i + \frac{(d_o - d_i)^2}{3} \right) \quad (2)$$

$$\approx \frac{\pi}{4} \left(\frac{d_i + d_o}{2} \right)^2 = \frac{\pi}{4} d^2$$

as long as the surround is not wider than 10 % of the outer diameter d_o and the assumption in Eq. (1) is valid.

Significant higher errors are found in woofers using a larger surround designed for realizing high peak displacement X_{\max} and especially in tweeters, headphones, horn compression drivers and microspeakers where the outer part of the diaphragm itself is used as suspension system.

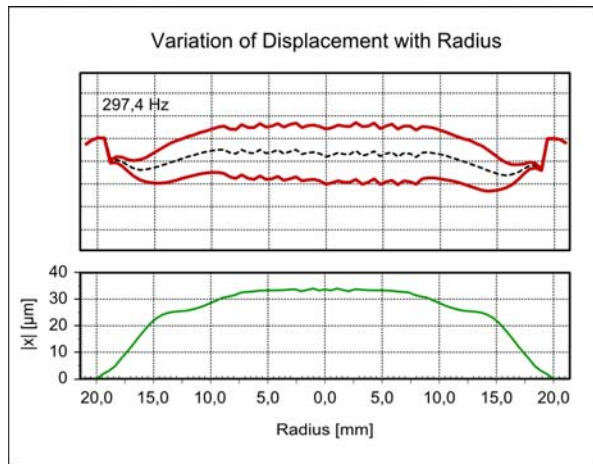


Fig. 2: Nonlinear distribution of cone displacement for a headphone driver

In those cases the displacement rises from zero at the outer rim to the maximal value at the voice coil position by a function which is usually nonlinear (see Fig. 2 above and Fig. 8 in [5] and depends on the particular clamping condition of the rim, the thickness and the curvature of the diaphragm profile and the material properties. Theoretical estimations of effective radiation area S_D for clamped plates, flat circular membranes and other simple geometries are given in [6] but may be not applicable to real transducers having a much more complicated geometry. Thus the precise measurement of the effective radiation area S_D has to be calculated from the total volume velocity $q(\omega)$ measured by using mechanical and/or acoustical sensors [7]-[9].

The paper copes with this problem and searches for simple, robust and convenient measurement techniques giving more reliable values of S_D . At the beginning the effective radiation area S_D will be defined by using the sound pressure output predicted by the Rayleigh equation. In the second part the pro and cons of the traditional techniques are discussed. A new technique is presented which calculates the S_D from the vibration and radiator's geometry using the results of modern laser scanning techniques. The accuracy of the new method is investigated and compared with traditional methods.

2. SYMBOLS

\mathbf{r}_a	observing point in the far field
\mathbf{r}_c	point on the radiator's surface
r_0	distance between center of the radiator and observing point \mathbf{r}_a
ω	angular frequency
ω_0	angular resonance frequency
$\underline{p}(\omega, \mathbf{r}_a)$	complex sound pressure at observing point in the far field
$\underline{p}_{box}(\omega)$	complex sound pressure in the sealed test enclosure
$\underline{x}(\omega, \mathbf{r}_c)$	complex displacement at the radiator's surface
$\underline{x}_{coil}(\omega)$	complex voice coil displacement
$\underline{S}_D(\omega)$	complex value of the effective radiation area depending on frequency
S_D	effective radiation area describing the piston mode at resonance
ρ_0	density of air
S_c	total radiator's surface
k	wave number
r_{avg}	averaging radius (usually at voice coil position)
r_r	outer radius of the radiator
$\underline{q}(\omega)$	total volume velocity generated by the radiator
C_{ab}	acoustical compliance of the air volume in the sealed enclosure
ΔV_{box}	variation of the air volume in the test enclosure
V_{box}	total air volume of the test enclosure
p_0	static air pressure
κ	adiabatic index
$\underline{x}_{coil+\Delta V}(\omega_0)$	voice coil displacement at resonance frequency measured in the box with enlarged air volume

$\underline{p}_{box+\Delta V}(\omega_0)$ sound pressure at resonance frequency measured in the box with enlarged air volume
 d_i inner diameter of the surround area
 d_o outer diameter of the surround area
 d diameter referred to the middle of the surround area

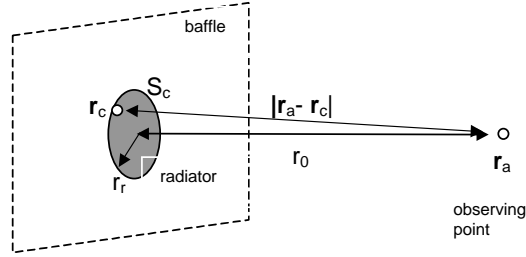


Fig. 3: Sound pressure generated on axis at observing point \mathbf{r}_a by a radiator mounted in an infinite baffle.

3. DEFINITION OF S_D

A radiator mounted in baffle generates a sound pressure

$$\underline{p}(\omega, \mathbf{r}_a) = -\frac{\omega^2 \rho_0}{2\pi} \int_{S_c} \frac{e^{-jk|\mathbf{r}_a - \mathbf{r}_c|}}{|\mathbf{r}_a - \mathbf{r}_c|} \underline{x}(\omega, \mathbf{r}_c) dS \quad (3)$$

at a listening point \mathbf{r}_a in the far field using the Rayleigh Integral of complex displacement $\underline{x}(\omega, \mathbf{r}_c)$ over all points \mathbf{r}_c on the surface S_c of the radiator as illustrated in Fig. 3.

If the listening point \mathbf{r}_a is on axis, far away from the radiator and much larger than the diameter $2r_r$ of the radiator then the distance $|\mathbf{r}_a - \mathbf{r}_c| \approx r_0$ may be assumed as a constant and Eq. (3) simplifies to

$$\underline{p}(\omega, \mathbf{r}_a) = -\frac{\omega^2 \rho_0}{2\pi} \frac{e^{-jkr_0}}{r_0} \int_{S_c} \underline{x}(\omega, \mathbf{r}_c) dS \quad (4)$$

The same sound pressure

$$\underline{p}(\omega, \mathbf{r}_a) = \frac{j\omega \rho_0}{2\pi} \frac{e^{-jkr_0}}{r_0} \underline{q}(\omega) \quad (5)$$

is generated by a volume velocity

$$\underline{q}(\omega) = j\omega \int_{S_c} \underline{x}(\omega, \mathbf{r}_c) dS \quad (6)$$

emitted by a point source in the center of the radiator.

An equivalent volume velocity

$$\underline{q}(\omega) = j\omega \underline{x}_{coil}(\omega) \underline{S}_D(\omega) \quad (7)$$

can be generated by a piston vibrating with the reference displacement $\underline{x}_{coil}(\omega)$ and an effective radiation area $\underline{S}_D(\omega)$.

Combining Eqs. (6) and (7) gives the effective radiation area

$$\underline{S}_D(\omega) = \frac{\underline{q}(\omega)}{j\omega \underline{x}_{coil}(\omega)} = \frac{\int_{S_c} \underline{x}(\omega, \mathbf{r}_c) dS}{\underline{x}_{coil}(\omega)} \quad (8)$$

which is a complex quantity and depends on frequency.

At low frequencies where all parts of the radiator vibrate in phase the complex function $\underline{S}_D(\omega)$ becomes a real value and the effective radiation area can be defined as

$$S_D = |\underline{S}_D(\omega_0)| \quad (9)$$

using the fundamental resonance frequency ω_0 as a useful reference.

4. MEASUREMENT TECHNIQUES

The following dynamic measurement techniques make no assumption on the displacement $\underline{x}(\omega, \mathbf{r}_c)$ on the radiator's surface.

4.1. Far-field Measurement in Baffle

Using Eqs. (5) and (7) gives the frequency dependent function

$$\underline{S}_D(\omega) = -\frac{2\pi}{\omega^2 \rho_0} \frac{r_0}{e^{-jkr_0}} \frac{\underline{p}(\omega, \mathbf{r}_a)}{\underline{x}_{coil}(\omega)} \quad (10)$$

which becomes the effective radiation area

$$S_D = |S_D(\omega_0)| = \frac{2\pi r_0}{\omega_0^2 \rho_0} \frac{|p(\omega_0, r_a)|}{|x_{coil}(\omega_0)|} \quad (11)$$

at the resonance frequency according to Eq. (9). This measurement technique requires a calibrated microphone for measuring the sound pressure $p(\omega, r_a)$ on axis at a distance r_0 from the radiator mounted in a baffle. The voice coil displacement is measured at multiple points on the surface and averaged to compensate for non axial-symmetrical vibration modes. This technique requires a sufficiently large baffle and an anechoic environment at low frequencies.

4.2. Near-field Measurement in Baffle

Moreno et. al. suggested in [5] a measurement of the sound pressure $p(\omega, r_a \approx r_c)$ close to the surface of the radiator to relax the requirements put on the acoustical environment and calculates the S_D by

$$S_D = |S_D(\omega_0)| = \frac{\pi}{4} \left(\frac{2}{\omega_0^2 \rho_0} \frac{|p(\omega_0, r_a)|}{|x_{coil}(\omega_0)|} \right)^2 \quad (12)$$

However, Moreno [5] observed a significant influence of the radiator’s geometry and measured a difference of more than 10 % in the S_D value for flat and conical pistons of the same size.

4.3. Box Method

A radiator mounted on a small enclosure as shown in Fig. 4 generates a sound pressure

$$\underline{p}_{box}(\omega) = \frac{q(\omega)}{j\omega C_{ab}} \quad (13)$$

which is at low frequencies almost independent of the microphone position [9].

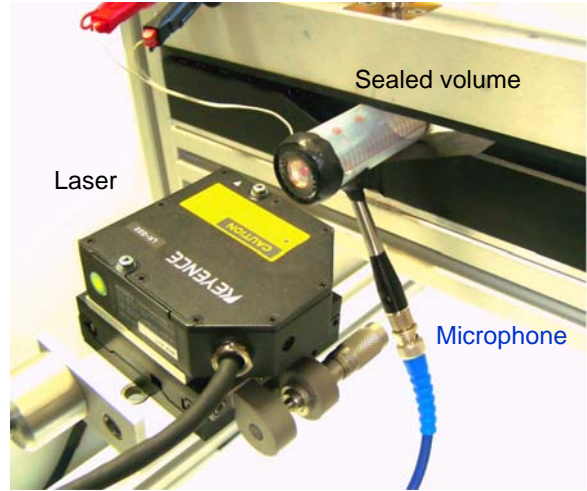


Fig. 4: Measurement of internal pressure generated in a sealed enclosure.

Calculating the acoustical compliance of the enclosed air

$$C_{ab} = \frac{V_{box}}{\kappa p_0} \quad (14)$$

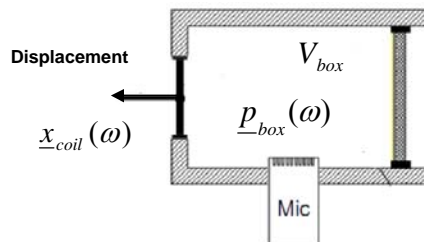
by using the air volume V_{box} , the adiabatic index κ and the static air pressure p_0 gives equivalent radiation area

$$S_D = |S_D(\omega_0)| = \frac{V_{box}}{\kappa p_0} \frac{|p_{box}(\omega_0)|}{|x_{coil}(\omega_0)|} \quad (15)$$

The volume V_{box} of the air in the test enclosure has to be measured carefully considering the free volume of the enclosure and the air displaced by the loudspeaker parts. In practice the determination of V_{box} limits the accuracy of the measurement technique.

4.4. Differential Box Method

Performing two measurements with different air volumes as illustrated in Fig. 5 dispenses with the measurement of static air volume displaced by the transducers.



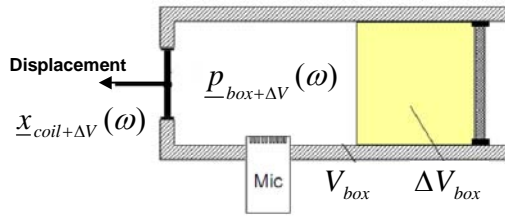


Fig. 5: Performing two measurements while using two test enclosures which have a precise difference in the air volume ΔV_{box}

Combining Eq. (15) of the first measurement with

$$S_D = \frac{V_{box} + \Delta V_{box}}{\kappa p_0} \frac{|p_{box+\Delta V}(\omega_0)|}{|x_{coil+\Delta V}(\omega_0)|} \quad (16)$$

of the second measurement and substituting V_{box} in both equations gives

$$S_D = \frac{1}{\kappa p_0} \frac{\Delta V_{box}}{\frac{|x_{coil+\Delta V}(\omega_0)|}{|p_{box+\Delta V}(\omega_0)|} - \frac{|x_{coil}(\omega_0)|}{|p_{box}(\omega_0)|}} \quad (17)$$



Fig. 6: Performing a differential measurement of the effective radiation area of a headphone by using a syringe.

A medical injection pump as shown in Fig. 6 may be used as an test enclosure for microspeakers, headphones and tweeters which allows a precise variation of the box

volume ΔV_{box} . Although the differential box puts the lowest requirement on the acoustical environment the major disadvantage of this technique is that a minor leakage may cause significant error in the measurement.

4.5. Laser Technique

A different approach for determining S_D can be based on the relations shown in equations (8) and (9). There S_D is defined as the integrated displacement of the radiator surface divided by the voice coil excursion $x_{coil}(\omega)$.

The displacement can be measured by a triangulation laser with very high accuracy. An automatic scanning tool can be used to measure the displacement $x(\omega, r_{c,i})$ at many points i on the radiator surface.

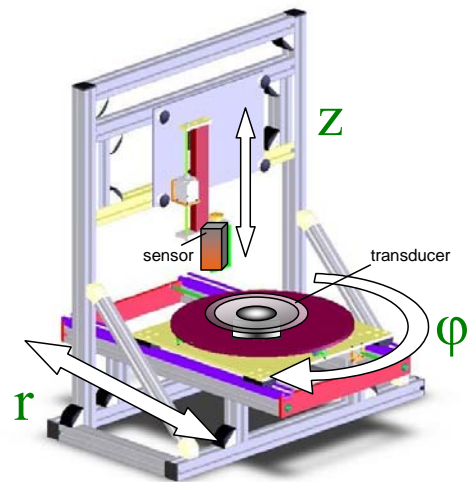


Fig. 7: Laser scanning technique for measuring vibration and geometry of the radiator's surface

The surface integration is done by multiplying the displacement at each point i with the respective surface element $\Delta S_{c,i}$ surrounding it.

$$S_D(\omega) = \frac{\sum x(\omega, r_{c,i}) \cdot \Delta S_{c,i}}{x_{coil}(\omega)} \quad (18)$$

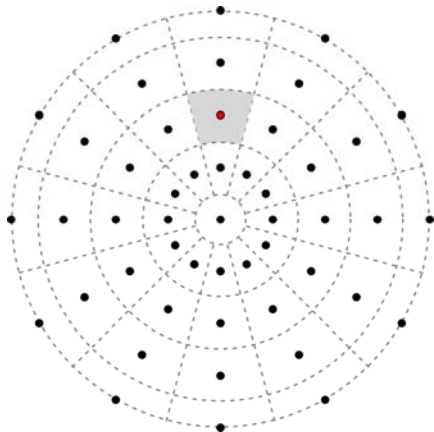


Fig. 8: Cone surface elements $\Delta S_{c,i}$

For the measurement of the coil excursion $x_{coil}(\omega)$ it is advisable to measure not only one single point but rather conduct a spatial averaging by measuring multiple points e.g. on a circle close to the voice coil.

$$\underline{x}_{coil}(\omega) = \frac{\int_0^{2\pi} \underline{x}(\omega, r_{avg}, \varphi) d\varphi}{2\pi} \tag{19}$$

The averaging radius r_{avg} should be selected close to the voice coil to be representative for the coil motion in piston mode.

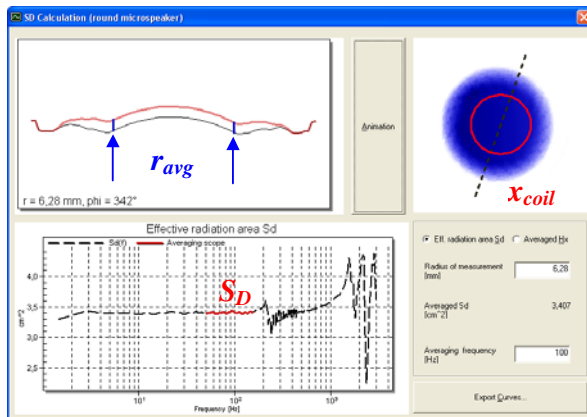


Fig. 9: User interface of the S_D calculation tool required to specify the voice coil position

4.5.1. Frequency Dependency of $\underline{S}_D(\omega)$

By applying Eq. (18) a frequency dependent effective radiation area $\underline{S}_D(\omega)$ is obtained. At low frequencies below the break-up of the cone the amplitude of $\underline{S}_D(\omega)$ is constant and represents the effective radiation area of the piston motion which is equal to the T/S parameter S_D , compare Eq. (9).

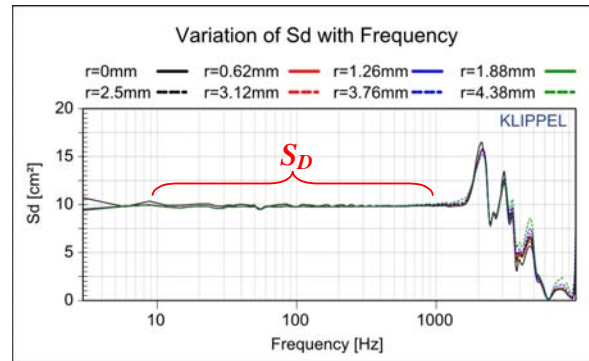


Fig. 10: Effective radiation area $|\underline{S}_D(\omega)|$ versus frequency using 8 different values of r_{avg} for averaging the voice coil displacement.

The graphs of $|\underline{S}_D(\omega)|$ are shown in Fig. 10 for a set of different r_{avg} in the center of an oval microspeaker. Over a broad frequency range from 10Hz to 1kHz the derived S_D is nearly constant. An averaging in that frequency range will further improve the accuracy of the nominal S_D of the driver.

At higher frequencies $\underline{S}_D(\omega)$ is not constant anymore but depends on the specific vibration properties at the averaging radius r_{avg} in relation to the total volume flow $\underline{q}(\omega)$.

As long as the used r_{avg} is close to the voice coil of the driver, the respective $\underline{S}_D(\omega)$ represents a linear transfer function between a given coil displacement $\underline{x}_{coil}(\omega)$ and the far-field sound pressure on axis for, see Eq. (20) which can be derived from Eq. (10).

$$\underline{p}(\omega, r_0) = -\frac{\omega^2 \rho_0}{2\pi} \frac{e^{-jk r_0}}{r_0} \underline{S}_D(\omega) \cdot \underline{x}_{coil}(\omega) \tag{20}$$

4.5.2. Averaging Radius r_{avg}

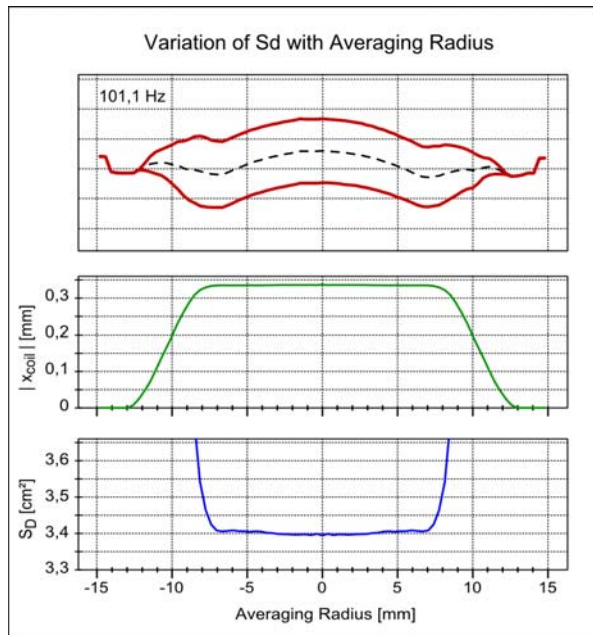


Fig. 11: Cross sectional view of the radiator with enhanced peak and bottom displacement (top), displacement versus radius (middle) and effective radiation area S_D versus r_{avg} at 100Hz.

If r_{avg} is too far away from the voice coil then the coil excursion $x_{coil}(\omega)$ will obtain an error which leads to an incorrect S_D .

The measured graphs for x_{coil} and S_D are shown in Fig. 11 depending on the location of the averaging radius r_{avg} and for a frequency of 100Hz. It can be seen that the amplitude of the coil displacement is nearly constant in the center of the driver up to a radius of 7mm and then it drops to zero until the outer driver edge. Likewise the derived S_D is valid up to a radius of 7mm.

4.5.3. Measurement grid resolution

The resolution of the scanning grid for measuring the displacement $\underline{x}(\omega, r_{c,i})$ is important for the accuracy of the determined S_D .

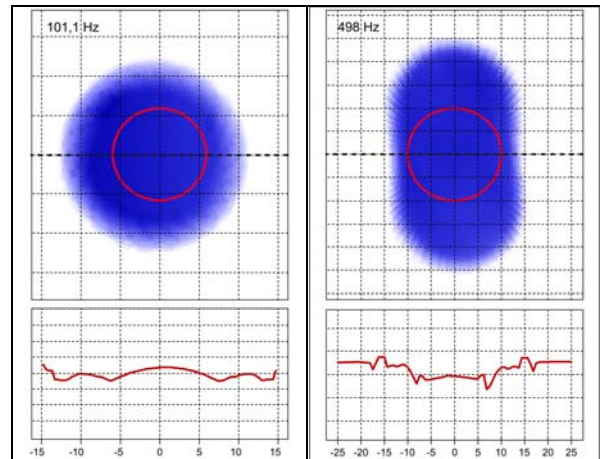


Fig. 12: Intensity plot of the cone vibration (top) and sectional view (bottom) of a circular and oval micro-speaker.

An investigation of the effect of reducing the number of radial and angular measurement points was conducted for the two microspeakers shown in Fig. 12.

In both cases a full detailed scan of 3201 points was coarsened by removing the data from single measurement radii or angles. The averaging radius has been set to 6mm for the circular speaker and to 10mm for the oval speaker.

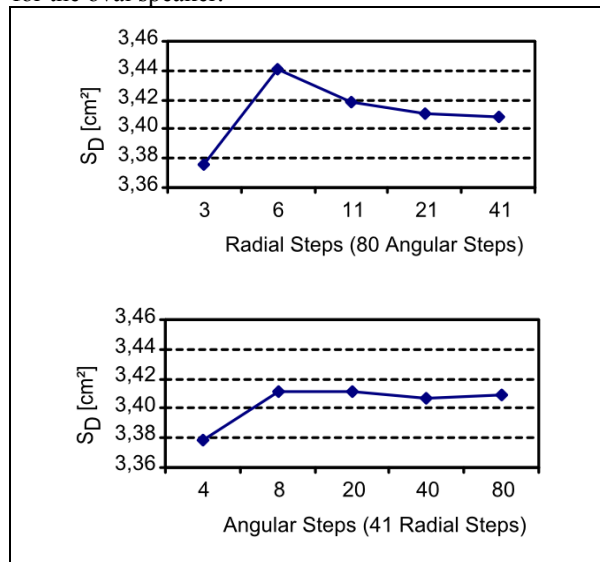


Fig. 13: Calculated values of effective radiation area S_D versus radial grid resolution (top) and angular resolution (bottom) for a circular speaker.

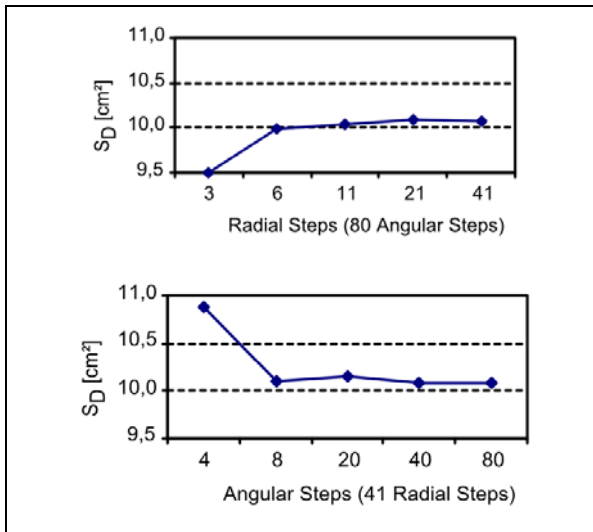


Fig. 14: Calculated values of effective radiation area S_D versus radial grid resolution (top) and angular resolution (bottom) for an oval speaker.

The nominal value for S_D remains very accurate for the circular speaker, even when only three radial lines or likewise four angular lines were kept. Even a combination of three radial and four angular steps will result in $S_D = 3,375\text{cm}^2$ which is only 1% away from the full resolution result of $S_D = 3,408\text{cm}^2$.

That means in this case with the measured $\underline{x}(\omega, r_{c,i})$ at only 9 points it is possible to determine the S_D of the driver with acceptable accuracy. But it should be stated that this result can not be applied to all circular speakers. In general at least a higher radial resolution is necessary to cover properly the decay of the displacement amplitude towards larger radii. As long as the driver shape is axis-symmetrical, a higher angular resolution will only improve the signal to noise ratio of the measurement and can help to evaluate the influence of rocking modes correctly.

For the oval speaker the error becomes larger if too few radial or angular steps are considered. Especially a higher angular resolution is here necessary to measure the displacement distribution on an asymmetrical speaker with sufficient accuracy.

The measured cone displacement data should cover the complete driver area to give a proper value for S_D by applying Eq. (18). But sometimes that is not possible e.g. when certain cone regions are not accessible by the laser because they are hidden behind a cover. Then the

integrated surface displacement will be too low which leads to an S_D which is also too low.

Furthermore the triangulation laser can produce errors while measuring the vibration at sharp edges or poorly reflective spots on the cone. These erroneous measurement points will also affect the calculation of S_D . The scanning software used for the present examples can recognize erroneous points by correlating two subsequent measurements from different distances. These points will be removed from any further analysis by setting the measured displacement to zero. Again the integrated surface displacement will be lower depending on the number and location of the erroneous points.

5. DISCUSSION

The laser technique proposed in section 4.5 performs a vibration measurement of the complete cone. Measuring the displacement at several points instead of only one point is also advisable for the single box technique in section 4.3 and the differential box technique in section 4.4. The reason is that especially small speakers with no properly centering suspension are susceptible to asymmetrical vibrations and rocking modes as shown in Fig. 15.

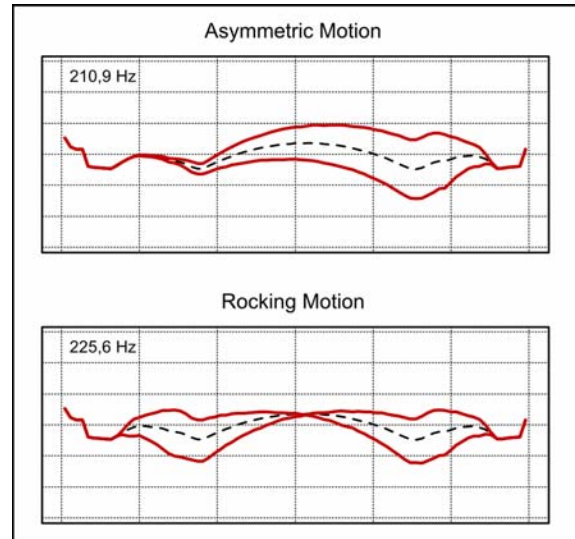


Fig. 15: Irregular cone vibration

These vibration modes usually occur at low frequencies where the speaker is supposed to perform a piston motion. The displacement measured at only one point will not be representative for the motion of the complete cone in presence of asymmetrical vibration modes.

Therefore it is necessary to perform a set of vibration measurements at selected points on the cone and average the measured data.

6. CONCLUSION

An accurate measurement of the effective radiation area S_D is the basis for a reliable prediction of the acoustical output. An error of 12 % in S_D may cause an error of 1 dB in pass band sensitivity.

Exploiting the geometry of the radiator only by reading the diameter in the middle of the surround is the most simple technique which may be applicable to woofers where the width of the surround is small compared to total diameter of the radiator and the total volume velocity is generated by the cone vibrating as a rigid body. This technique is not reliable for headphones, microspeakers, compression drives and other transducers where the vibration varies significantly versus the radiator's surface. Here the total volume velocity has to be measured using acoustical and/or optical sensors. The far field measurement suggested in the paper is a simple technique giving more accurate results than the near field measurement but requires a large baffle and an anechoic environment. The differential box technique puts lower requirements on the test equipment but requires a careful sealing of the test enclosure.

The measurement of the voice coil displacement requires a mechanical measurement which can be accomplished by optical laser triangulation sensors. The measurement of multiple points is required to consider non axial-symmetrical modes as found on most transducers dispensing with a spider.

If the radiator's surface is completely accessible to an optical laser scanner no acoustical measurements are required and a most precise value of the effective radiation area can be calculated from the measured vibration and geometry.

7. REFERENCES

- [1] Leo Beranek, "Acoustics", Acoustical Society of America, pp. 231
- [2] R. H. Small, "Direct-Radiator Loudspeaker System Analysis," J. of Audio Eng. Soc., Volume 20, pp. 383 – 395 (1972 June).
- [3] D. Clark, "Precision Measurement of Loudspeaker Parameters", J. Audio Eng. Society 45, No. 3 pp. 129 - 141 (1997 March).
- [4] W. Klippel, U. Seidel, "Fast and Accurate Measurement of Linear Transducer Parameters" presented at the 110 Convention of the Audio Engineering Society, 2001 May, Amsterdam.
- [5] J. N. Moreno, R. A. Moscoso, S. Johnsson, "Measurement of the Effective Radiating Surface Area of a Loudspeaker Using a Laser Velocity Transducer and a Microphone" presented at the 96th Convention of the Audio Engineering Society, 1994 February, Amsterdam, preprint 3861.
- [6] R. Ballas, G. Pfeifer, R. Werthschützky, "Elektromechanische Systeme der Mikrotechnik und Mechatronik", Springer Verlag Berlin Heidelberg 2009.
- [7] J. N. Moreno, S. Johnsson, Henning Bog "Measurement of Closed Box Loudspeaker System Parameters using a laser Velocity Transducer and an Audio Analyzer," presented at the 92nd Convention of the Audio Engineering Society, 1992 March, Preprint 3325.
- [8] J. N. Moreno, "Measurement of Loudspeaker Parameters Using a Laser Velocity Transducer and Two Channel FFT Analysis," J. Audio Eng. Soc., Vol. 39 (1991 April).
- [9] D. K. Anthony and S. J. Elliot, "A Comparison of three Methods of Measuring the Volume Velocity of an Acoustic Source," J. Audio Eng. Soc., Vol. 39, pp. 355-366 (1991 May)